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Aplicação de Modelos Teóricos de Grafos na Análise de Sistemas de Potência.

Dissertação de Mestrado em
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ABSTRACT

This work attempts to present an application of graph theory to solve some problems on power system analysis such as: short circuit, load flow and transient stability studies, using digital methods. The applications are: i) a simulation of the CHESF system (Brazilian Power System) for short circuit and load flow studies, ii) a simulation for short circuit of a system where there is ungrounded generator and iii) transient stability simulation for a system where there are several machines.

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PART ONE

PART TWO

INTRODUCTION

I. The use of graph theory for the formulation and solution of problems on power system analysis has become very important with the advent of digital computers. Here this fact is made use of to solve some complex problems in power systems by simulation.

II. Simulation of the CHESF System for short circuit and load flow.

II.1 Short circuit simulation - For short circuit simulation we will show how to find a terminal equation in impedance form for the system using Z_{BUS} formulation. Three phase to ground fault and single phase to ground fault are considered giving detailed computational methods.

II.2 Load flow simulation - In this case we will obtain a terminal equation in admittance form for the system. Using this a set of non linear equations are solved by several iterative methods such as: Gauss-Seidel and Newton Raphson. As an application of load flow, a simple study of the regulation of the CHESF System is undertaken.

III. Short circuit simulation for a system in the presence of ungrounded generators - Here we will simulate a small system (given in the book "Power System Stability" by Kimbark) for single line to ground fault. There are two generators in the system, where one of them has isolated neutral; Z_{BUS} formulation is used for this purpose.

IV. <u>Transient stability simulation</u> - The purpose of this simulation is to obtain a plot of the swing curve for a system when a three phase to ground fault occurs at one bus of the system. The particular system used here is given in the paper "Transient Stability Regions of Multimachine Power System" by A. H. El-Abiad and K. Nagappan.

PART ONE

1. CHESF SYSTEM

1.1 SOME CONSIDERATIONS

Our study will be done only with the primary system, i.e. the transmission system. We will assume only the main thirteens bus of the system, taking into account its loads and its distances from the generator bus.

From the above considerations, the system buses will be:

1 - PAULO AFONSO
2 - CATU
3 - ITABAIANA
4 - BOM NOME
5 - MILAGRES
6 - BANABUIU
7 - FORTALEZA
8 - MATATU
9 - ANGELIN
10 - CAMPINA GRANDE
11 - SANTA CRUZ
12 - RECIFE
13 - GOIANINHA

1.2 OBSERVATIONS

This study does not take into account the further modifications foreseen by CHESF Directory. It will assume the situation of the system as in September, 1971.

1.3 CONSIDERATIONS OF THE SINGLE LINE DIAGRAM

We will assume power generation only in the bus 1 (PAULO AFONSO) because the others are much smaller than it. The rest of the buses will be load buses. Also we will assume initially that all transformers are in nominal tap-settings. As each chosen bus has one distribution system belonging to it, the load of each bus will be the sum of the loads of all small buses of the distribution system, neglecting its subtransmission lines. The load in the single line diagram is represented by one arrow. At some buses of the system, there are reactors (shunt reactors), whereas some of them are directly on the high voltage side which will appear in the single line diagram. The single line diagram is shown in Figure 1-1.

1.4 SYSTEM DATA

1.4.1 PER UNIT VALUES

All the values of powers, voltages, currents, impedances, admittances, etc. will be written in per unit values, using the following base:

POWER - 100 MVA

VOLTAGE - 230 KV at the lines where the nominal voltage is 230 KV
and 138 KV at the lines where the nominal voltage is 138 KV.

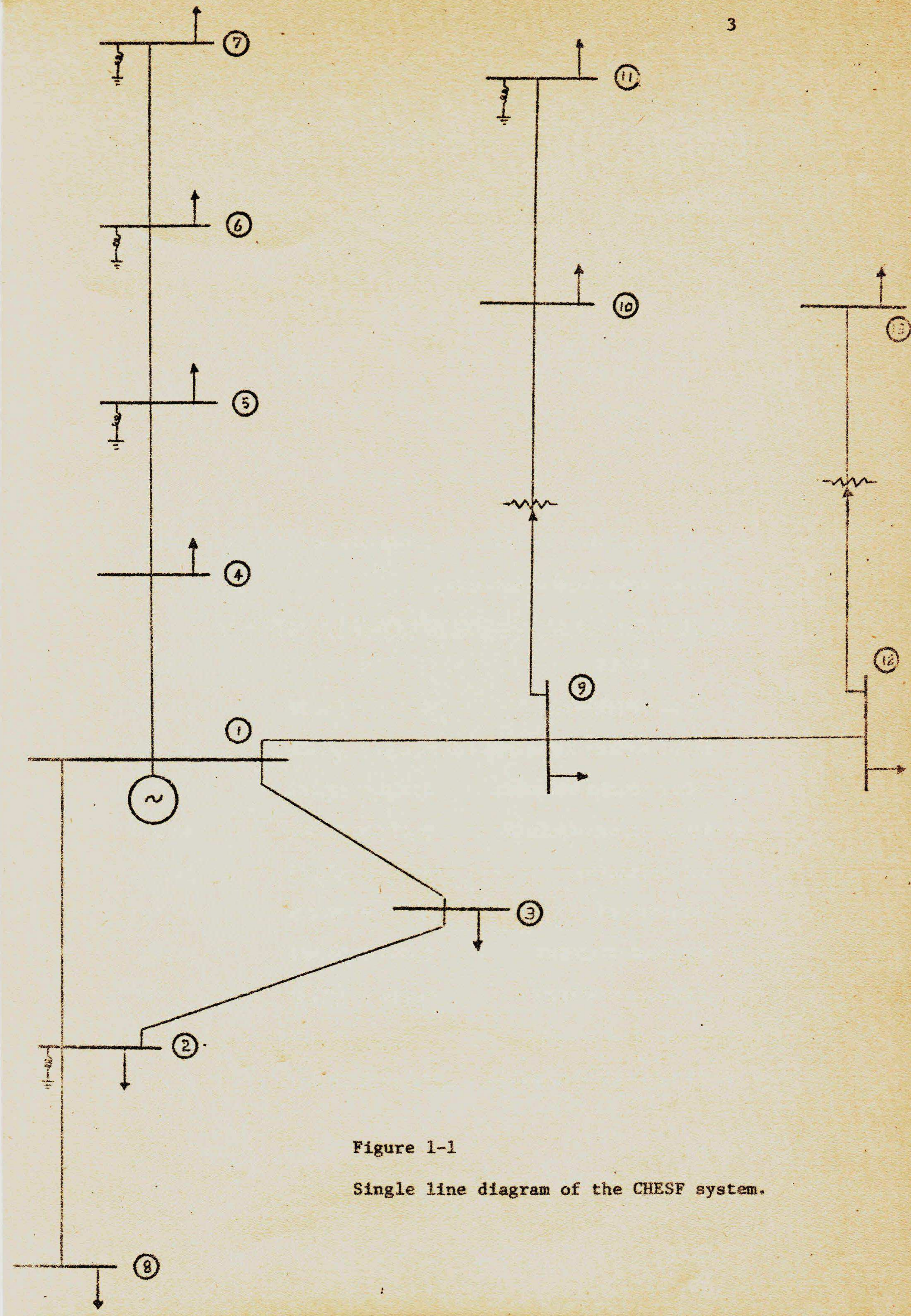


Figure 1-1

Single line diagram of the CHESF system.

1.4.2 TRANSMISSION LINES

As all of the transmission lines are "long lines", we will represent them by π equivalent (see Figure 1-2).

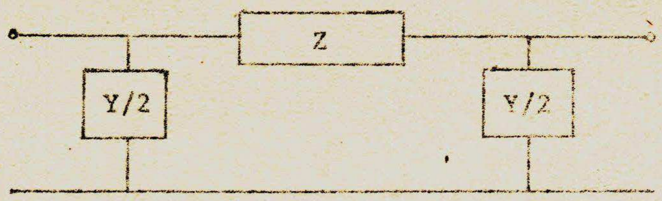


Figure 1-2

where Z is the total series line impedance and Y is the total shunt admittance, both in complex form.

VALUES OF Z AND Y/2 AT POSITIVE AND NEGATIVE SEQUENCES

	LINES	Z	Y/2
1-2	PAULO AFONSO-CATU	0.060 + j0.320	j0.282
1-3	PAULO AFONSO-ITABAIANA	0.030 + j0.162	j0.133
1-4	PAULO AFONSO-BOM NOME	0.032 + j0.163	j0.144
1-9	PAULO AFONSO-ANGELIN	0.012 + j0.064	j0.558
2-3	CATU-ITABAIANA	0.040 + j0.213	j0.176
2-8	CATU-MATATU	0.015 + j0.078	j0.063
4-5	BOM NOME-MILAGRES	0.016 + j0.081	j0.071
5-6	MILAGRES-BANABUIU	0.042 + j0.218	j0.188
6-7	BANABUIU-FORTALEZA	0.032 + j0.168	j0.143
9-10	ANGELIN-CAMPINA GRANDE	0.035 + j0.177	j0.158

9-12 ANGELIN-RECIFE	0.009 + j0.053	j0.464
10-11 CAMPINA GRANDE-SANTA CRUZ	0.049 + j0.148	j0.076
12-13 RECIFE-GOIANINHA	0.068 + j0.173	j0.022

VALUES OF Z AT ZERO SEQUENCE

LINES	Z
1-2 PAULO AFONSO-CATU	0.230 + j0.835
1-3 PAULO AFONSO-ITABAIANA	0.118 + j0.423
1-4 PAULO AFONSO-BOM NOME	0.134 + j0.446
1-9 PAULO AFONSO-ANGELIN	0.070 + j0.258
2-3 CATU-ITABAIANA	0.153 + j0.554
2-8 CATU-MATATU	0.058 + j0.203
4-5 BOM NOME-MILAGRES	0.068 + j0.223
5-6 MILAGRES-BANABUIU	0.166 + j0.580
6-7 BANABUIU-FORTALEZA	0.130 + j0.448
9-10 AGNELIN-CAMPINA GRANDE	0.145 + j0.486
9-12 ANGELIN-RECIFE	0.061 + j0.204
10-11 CAMPINA GRANDE-SANTA CRUZ	0.180 + j0.570
12-13 RECIFE-GOIANINHA	0.131 + j0.594

OBSERVATION

In the zero sequence calculations, the mutual couplings between the following sets of lines are also included: i) two parallel lines from CAMPINA GRANDE to SANTA CRUZ, ii) three parallel lines from PAULO AFONSO to ANGELIN and iii) three parallel lines from ANGELIN to RECIFE.

The equivalent impedance for negative sequence was determined as illustrated below:

Example: If we have two parallel lines with mutual coupling. Figure 1-3 shows the augmented graph.

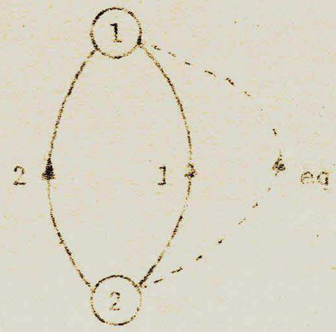


Figure 1-3

Fundamental circuit equations for the graph of Figure 1-3 with the element 2 as tree can be written

$$\begin{array}{c|ccc|} & 2 & 1 & \mathrm{eq} \\ 1 & -1 & 1 & 0 \\ \mathrm{eq} & -1 & 0 & 1 \end{array} \begin{bmatrix} V_2 \\ V_1 \\ V_{eq} \end{bmatrix} = 0 \tag{1-1}$$

or

$$\begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} \begin{bmatrix} V_2 \\ V_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_{eq} = 0 \tag{1-2}$$

Terminal equations:

$$\begin{bmatrix} V_2 \\ V_1 \end{bmatrix} = \begin{bmatrix} Z_{22} & Z_{12} \\ Z_{12} & Z_{11} \end{bmatrix} \begin{bmatrix} I_2 \\ I_1 \end{bmatrix} \qquad (1\text{-}3)$$

where Z_{22} and Z_{11} are self impedances and Z_{12} is mutual impedance. Substituting equation (1-3) into equation (1-2) we have:

$$\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} Z_{22} & Z_{12} \\ Z_{12} & Z_{11} \end{bmatrix} \begin{bmatrix} I_2 \\ I_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_{eq} = 0 \qquad (1\text{-}4)$$

Now, transforming the brench currents in terms of chord currents from the fundamental cut-set equation

$$\begin{bmatrix} I_2 \\ I_1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_{eq} \end{bmatrix} \qquad (1\text{-}5)$$

Substituting equation (1-5) into equation (1-4) we have:

$$\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} Z_{22} & Z_{12} \\ Z_{12} & Z_{11} \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_{eq} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_{eq} = 0$$

$$\begin{bmatrix} (Z_{22} + Z_{11} - 2Z_{12}) & (Z_{22} - Z_{12}) \\ (Z_{22} - Z_{12}) & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_{eq} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_{eq} \qquad (1\text{-}6)$$

From the top equation of equations (1-6) we can find the value of I_1 and substitute it into the second equation of equations (1-6). In this way, we will get one relation between V_{eq} and I_{eq}, and assuming that

$$V^o = V_{eq} \quad \text{and} \quad I^o = - I_{eq}$$

we have:

$$V^o = Z^o I^o$$

So, Z^o is the equivalent impedance for zero sequence, between two parallel lines with mutual coupling.

1.4.3 REACTORS DATA

The reactors directly on the bus (high voltage side) will be given by their admittances.

BUS	ADMITTANCE
2 - CATU	j0.10
5 - MILAGRES	j0.10
6 - BANABUIU	j0.10
7 - FORTALEZA	j0.20
11 - SANTA CRUZ	j0.05

Reactor specified in terms of load will be given by its power.

BUS	REACTIVE POWER
10 - CAMPINA GRANDE	0.10
5 - MILAGRES	0.30

1.4.4 STATIC CAPACITOR SPECIFIED IN TERMS OF LOAD

Will be given by its power.

BUS	REACTIVE POWER
3 - ITABAIANA	0.0881
6 - BANABUIU	0.0180
7 - FORTALEZA	0.2160
9 - ANGELIN	0.1020
10 - CAMPINA GRANDE	0.0540
11 - SANTA CRUZ	0.1277
12 - RECIFE	0.3980
13 - GOIANINHA	0.1980

1.4.5 LOADS

We have only the peak values of active powers for the system with respective load factors, which is the relation between the average load and the maximum load (peak value) for all buses (transmission and distribution systems).

BUS 2 - CATU

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - TAQUIPE	0.03834	0.152
2 - BURACICA	0.02112	0.547
3 - MATARIPE	0.04560	0.443
4 - FERBASA	0.11712	0.500
5 - FEIRA DE SANTANA	0.08256	0.400

6 - TIBRAS	0.02400	0.600
7 - COELBA	0.11448	0.700
8 - B.N-ARATU	0.06928	0.500
9 - CIA I	0.03200	0.500
10 - COTEGIPE	0.07600	0.425
11 - SIBRA	0.15800	0.600
12 - CATU	0.10704	0.623

BUS 3 - ITABAIANA

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - RIACHUELO	0.01180	0.454
2 - ARACAJU	0.16758	0.517
3 - S. CRISTOVAO	0.00720	0.307
4 - ESTANCIA	0.02880	0.434
5 - DORES	0.00936	0.315
6 - PROPRIA	0.01180	0.389
7 - CARRAPICHO	0.02880	0.423
8 - S. DIAS	0.00423	0.312
9 - P. VERDE	0.00816	0.304
10 - LAGARTO	0.01056	0.298
11 - CARMOPOLES	0.02280	0.400
12 - ATALAIA	0.03600	0.400
13 - ITABAIANA	0.01440	0.322

BUS 4 - BOM NOME

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - SALGUEIRO	0.00720	0.473
2 - CABROBO	0.01422	0.366
3 - S. TALHADA	0.00784	0.424
4 - PARNAMIRIM	0.01176	0.466
5 - BOM NOME	0.00288	0.365
6 - FLORES	0.01392	0.406

BUS 5 - MILAGRES

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - BALANCOS	0.00600	0.221
2 - JUAZEIRO	0.03168	0.458
3 - CRATO	0.02088	0.521
4 - CAJAZEIRAS	0.01279	0.600
5 - S. GONCALO	0.01279	0.305
6 - MALTA	0.00114	0.348
7 - POMBAL	0.00376	0.397
8 - JERICO	0.00720	0.282
9 - PATOS	0.01920	0.570
10 - CATOLE DO ROCHA	0.01248	0.401
11 - LAVRAS	0.00191	0.309
12 - CEDRO	0.00360	0.289
13 - IGUATU	0.01296	0.356

14 - OROS	0.00649	0.419
15 - CUREMAS	0.00202	0.375
16 - MILAGRES	0.01008	0.256

BUS 6 - BANABUIU

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - RUSSAS	0.00792	0.350
2 - S. POMPEU-JOATAMA	0.01440	0.350
3 - BANABUIU	0.00120	0.350

BUS 7 - FORTALEZA

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - FORTALEZA	0.03512	0.508
2 - CENORTE	0.12096	0.300
3 - CONEFOR	0.54720	0.400

BUS 8 - MATATU

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - MATATU	0.73000	0.505

BUS 9 - ANGELIN

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - GARANHUNS	0.04752	0.400
2 - CORRENTES	0.00350	0.317
3 - MATRIZ	0.01104	0.425
4 - UNIAO	0.01332	0.362
5 - ATALAIA	0.02496	0.371
6 - CAPELA	0.01440	0.361

7 - PILAR	0.00288	0.434
8 - S. MIGUEL	0.00756	0.393
9 - MACEIO	0.17280	0.485
10 - A. VERDE	0.02832	0.378
11 - PESQUEIRA	0.02520	0.382
12 - LAGEDO	0.01176	0.379
13 - CARUARU	0.07776	0.460
14 - BEZERROS	0.02400	0.439
15 - P. INDIOS	0.01391	0.377
16 - R. LARGO	0.02356	0.471
17 - ARAPIRACA	0.02064	0.351
18 - ANGELIN	0.00698	0.355

BUS 10 - CAMPINA GRANDE

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - CAMPINA GRANDE I	0.10656	0.484
2 - V. GRANDE	0.00780	0.485
3 - S.J. CARIRI	0.00696	0.287
4 - BOQUEIRAO	0.00848	0.733
5 - CAMPINA GRANDE II	0.02880	0.650

BUS 11 - SANTA CRUZ

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - NATAL	0.18576	0.501
2 - C. NOVOS	0.01816	0.397
3 - ACARI	0.02126	0.385

4 - S. MATEUS	0.00336	0.342
5 - ACU	0.00756	0.368
6 - MOSSORO	0.03240	0.479
7 - S.J. MIPIMBU-CEARAMIRIM	0.01800	0.353

BUS 12 - RECIFE

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - I. RETIRO	0.24840	0.534
2 - IMBIRIBEIRA	0.16560	0.500
3 - PIRAPAMA	0.09216	0.611
4 - CABO	0.01584	0.408
5 - JUCURAL	0.01152	0.389
6 - VITORIA	0.02640	0.455
7 - JABOATAO	0.08064	0.484
8 - S. LORENCO	0.04704	0.536
9 - CARPINA	0.02976	0.431
10 - LIMOEIRO	0.02982	0.416
11 - MACAXEIRA	0.16920	0.469
12 - BRENAND	0.02856	0.400
13 - POTY	0.05208	0.500
14 - PAULISTA	0.02304	0.450
15 - S. BENEDITO	0.15120	0.371
16 - J. MARIANO	0.06048	0.468
17 - MONJOPE	0.09393	0.561
18 - BONJI	0.85200	0.553
19 - PARATIPE	0.05576	0.400

BUS 13 - GOIANINHA

BUSES OF THE DISTRIBUTION SYSTEM	ACTIVE POWER	LOAD FACTOR
1 - GOIANA	0.01512	0.429
2 - MUSSURE	0.03528	0.327
3 - S. RITA	0.03660	0.454
4 - JOAO PESSOA	0.15840	0.450
5 - ORATORIO	0.00828	0.374
6 - TABAIANA	0.02664	0.384
7 - RIACHAO	0.00301	0.341
8 - RIO TINTO	0.02323	0.516
9 - AREIA	0.01620	0.405
10 - ACO NORTE	0.08148	0.700
11 - ITAPESSOCA	0.05376	0.400
12 - MARES	0.07104	0.595
13 - GOIANINHA	0.01694	0.368

Now, assuming that the power factors at all the buses are 0.9 (inductive) and the peaks of loads occur simultaneously at all the buses, we can find the equivalent load factor and total maximum power at each bus of the main system (transmission system) using the following expression.

$$LF_j = \frac{\sum_{i=1}^{n} S_i \, LF_i}{\sum_{i=1}^{n} S_i}$$

where LF_j = Load factor equivalent at the bus j of the main system.

LF_i = Load factor of the bus i of the distribution system.

n = Number of bus in each distribution system.

S_i = Apparent power of the bus i of the distribution system (maximum value).

$\sum_{i=1}^{n} S_i$ = Total maximum apparent power at the bus j of the main system.

$\sum_{i=1}^{n} S_i \, LF_i$ = Total average apparent power at the bus j of the main system.

Using the following program in WATF IV, we can easily obtain all those values above.

<u>PROGRAM</u>

```
$JOB      WATFIV
       COMPLEX S, SAV, SUM, SUMAV, CMPLX
       DO 12 K = 2, 13
       SUM = 0.0
       SUMAV = 0.0
       READ, M
       DO 11 J = 1, M
       READ, A, RF
       B = 0.487*A
       S = CMPLX (A,B)
       SUM = SUM + S
```

```
      SAV = S*RF
      SUMAV = SUMAV + SAV
   11 CONTINUE
      RF = SUMAV/SUM
      PRINT, 'BUS NO, K,' POWER = ', SUM,' AVG. POWER = ', SUMAV,'
                                                            LF', RF
   12 CONTINUE
      STOP
      END
$ENTRY
```

We can get from the output of the above program the following results:

BUS	MAXIMUM LOAD		AVERAGE LOAD		
	ACTIVE	REACTIVE	ACTIVE	REACTIVE	LOAD FACTOR
2 - CATU	0.8855	0.4313	0.4681	0.2280	0.529
3 - ITABAIANA	0.3615	0.1760	0.1615	0.0787	0.447
4 - BOM NOME	0.0578	0.0282	0.0241	0.0174	0.417
5 - MILAGRES	0.1650	0.0834	0.0704	0.0343	0.427
6 - BANABUIU	0.0235	0.0115	0.0082	0.0040	0.350
7 - FORTALEZA	0.7033	0.3425	0.2730	0.1329	0.388
8 - MATATU	0.7300	0.3555	0.3686	0.1795	0.505
9 - ANGELIN	0.5300	0.2581	0.2295	0.1118	0.433
10 - CAMPINA GRANDE	0.1586	0.0772	0.0823	0.0401	0.519
11 - SANTA CRUZ	0.2865	0.1395	0.1343	0.0654	0.469
12 - RECIFE	2.2353	1.0886	1.1427	0.5565	0.511
13 - GOIANINHA	0.5460	0.2659	0.2659	0.1295	0.487

1.4.6 TRANSFORMERS

FROM ANGELIN TO CAMPINA GRANDE

X = 0.365 off nominal taps as: 0.975, 0.950 0.925 and 0.900

nominal turn - 230 KV/138 KV YY

FROM RECIFE TO GOIANINHA

X = 0.322 off nominal taps as: 0.975, 0.950 0.925 and 0.900

nominal turn - 230 KV/138 KV YY

FROM THE GENERATOR TO BUS 1

X equivalent = 0.017

nominal turn - 13.8 KV/230 KV YY

1.4.7 GENERATOR

Set of ten generators in parallel with X^{11} equivalent = 0.031

Apparent power = 8.21.

2. SHORT CIRCUIT SIMULATION OF THE CHESF SYSTEM

2.1 INTRODUCTION

Our study of short circuit will be done only for balanced fault bus to ground; (This kind of fault is very rare in a system, but is a simple case of fault study. The purpose is to determine the capacity of the current for the circuit breaker of the buses of the system.) and phase to ground fault.

2.2 CONSIDERATIONS FOR SHORT CIRCUIT STUDY

2.2.1 TRANSMISSION SYSTEM

For the transmission system we will neglect:

1) The loads.

2) Line charging capacitances.

3) Other connections to ground reactors at the buses, etc.

Only the series impedance at positive sequence of each element is considered, and further, all transformers are assumed to be in nominal taps and its reactances will be included into the lines impedance.

2.2.2 GENERATOR SYSTEM

There is only one generator in the system under consideration. Therefore, the generator system will be represented by a voltage source in series with its internal impedance, which may be transient or subtransient reactance. In our case we will assume it to be subtransient reactance.

Figure 2-1 illustrates the single line diagram for short circuit simulation.

2.3 BALANCED FAULT; BUS TO GROUND USING Z_{BUS} REPRESENTATION

Our goal is to derive terminal equation of the form:

$$V_{BUS} = Z_{BUS} I_{BUS} + bV_o \qquad (2\text{-}1)$$

where V_{BUS} and I_{BUS} are the vectors representing the bus voltages and currents of the system. V_o is the matrix representing the active source due to generator and b is a column matrix.

From the equation (2-1) we establish the fault conditions and find short circuit currents at all buses of the system.

Initially we will derive the Z_{BN} matrix only for the transmission system corresponding to the terminal graph of Figure 2-3a.

For the augmented transmission system graph (Figure 2-4) which is the union of the transmission system graph with the augmented source drivers, we will write the fundamental circuit equations, choosing the elements (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13) as tree.

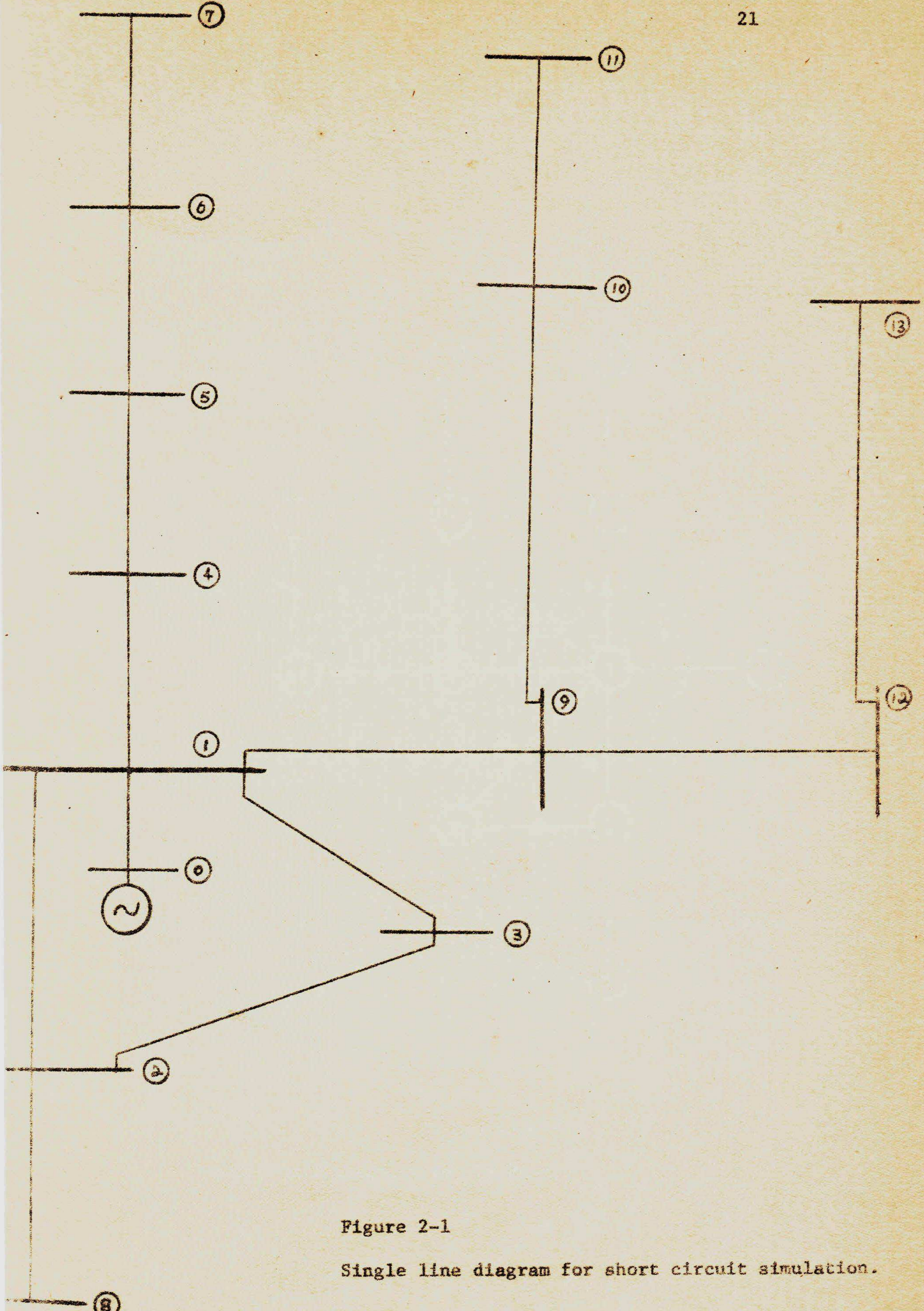


Figure 2-1

Single line diagram for short circuit simulation.

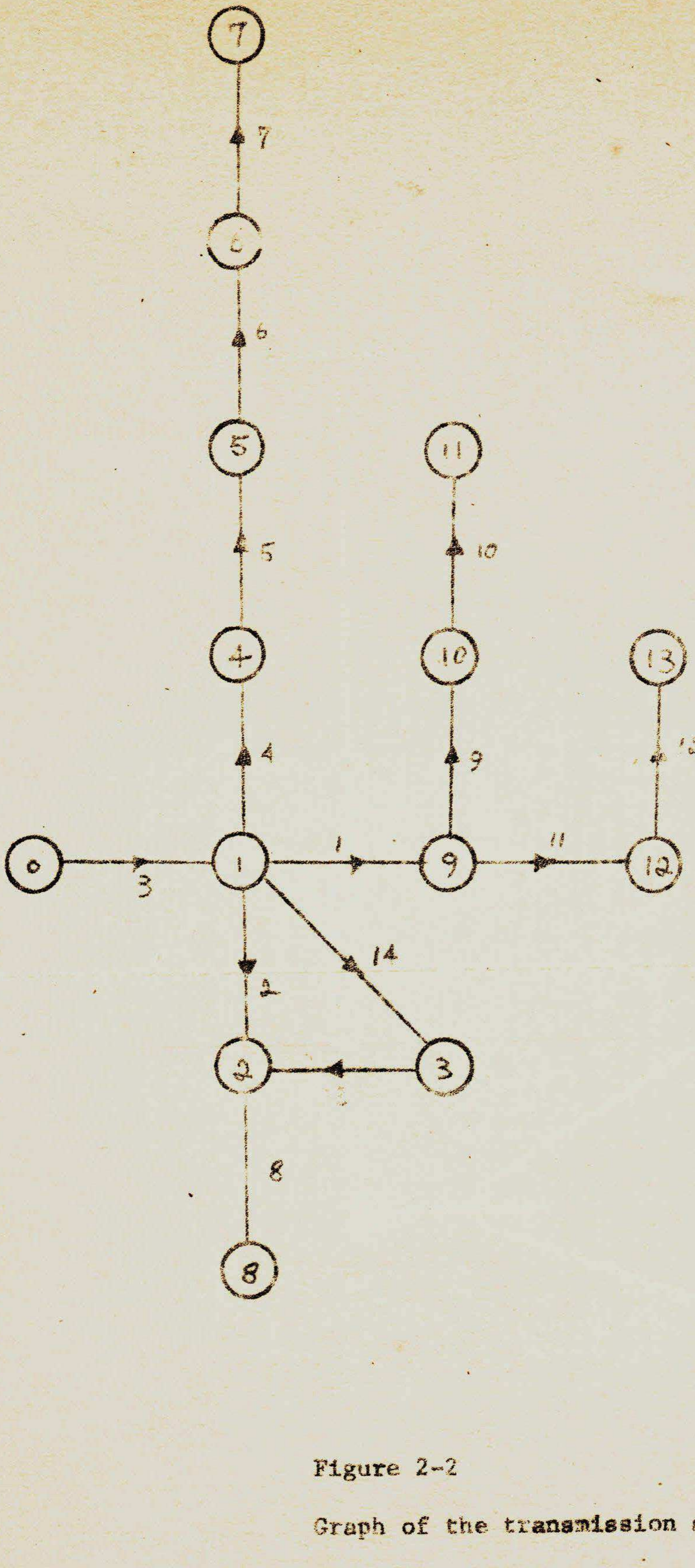


Figure 2-2

Graph of the transmission system.

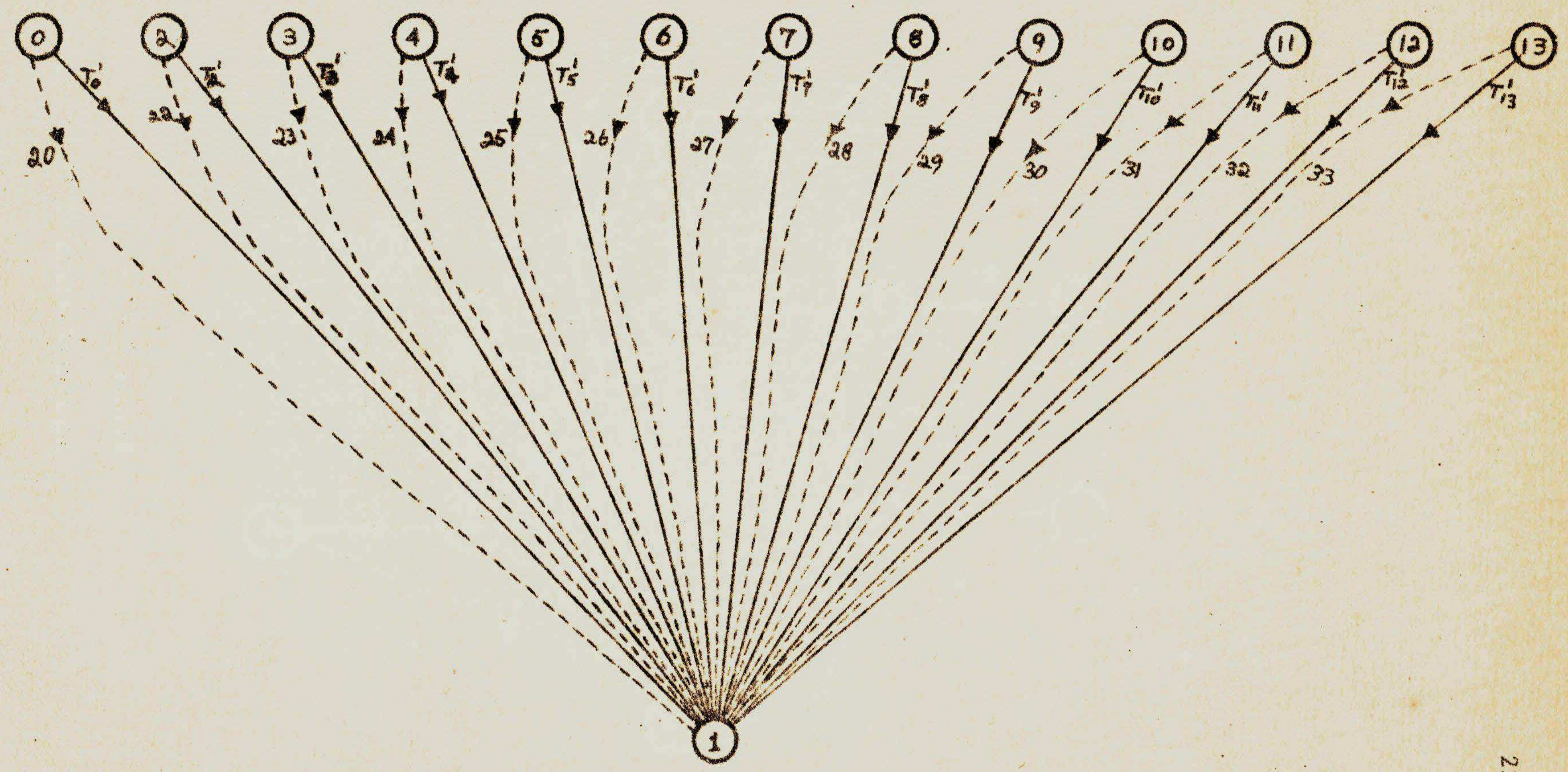


Figure 2-3

a) Solid lines - terminal graph of the transmission system.

b) Dashed lines - augmented current drivers.

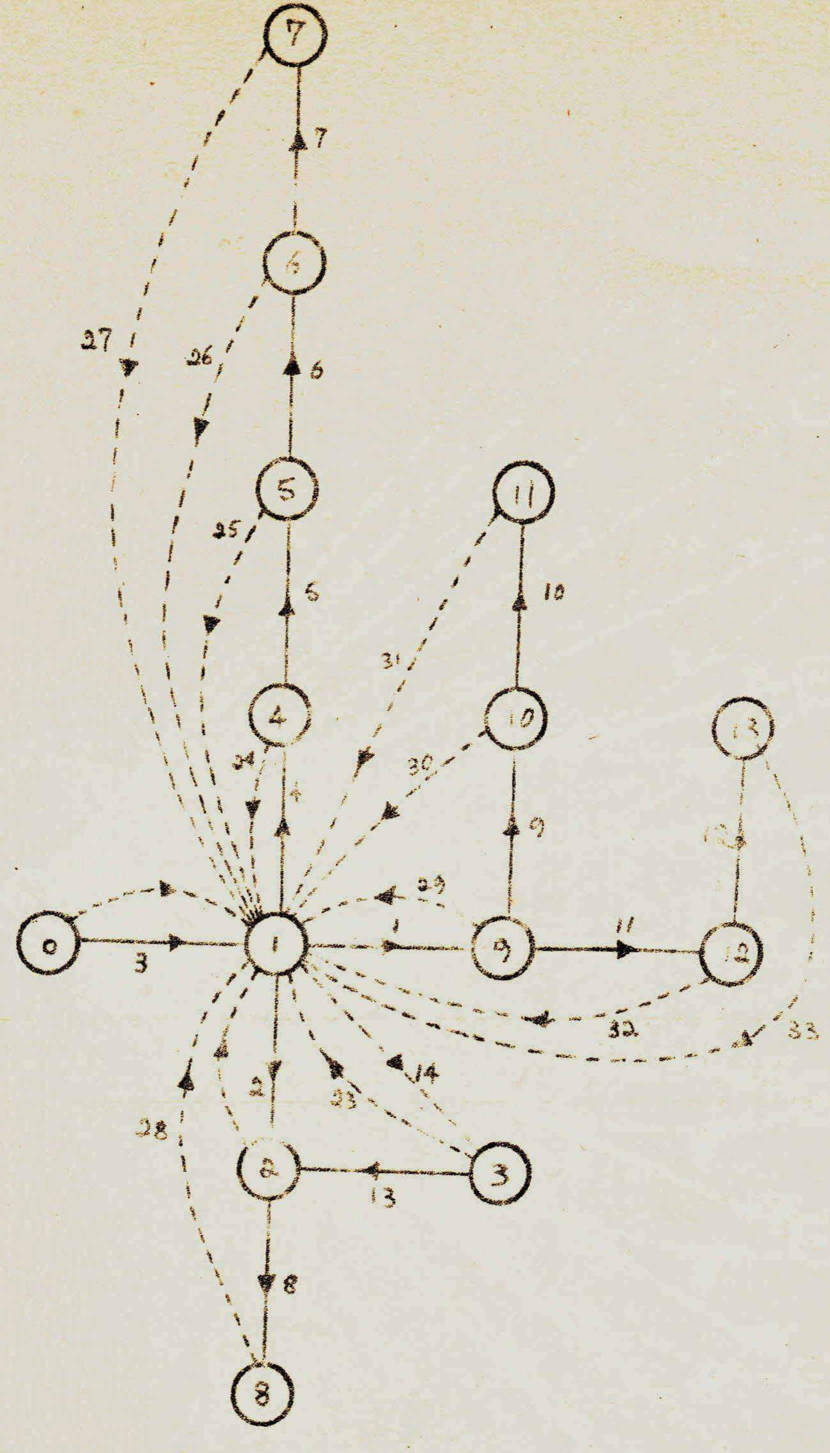


Figure 2-4

Augmented graph of the transmission system.

$$
\begin{array}{c}
14\\ 20\\ 22\\ 23\\ 24\\ 25\\ 26\\ 27\\ 28\\ 29\\ 30\\ 31\\ 32\\ 33
\end{array}
\underbrace{\left[\begin{array}{ccccccccccccc|c}
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1\\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{array}\right|}_{\underline{B}_1}
\left.\begin{array}{ccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\left[\begin{array}{c}
V_1\\ V_2\\ V_3\\ V_4\\ V_5\\ V_6\\ V_7\\ V_8\\ V_9\\ V_{10}\\ V_{11}\\ V_{12}\\ V_{13}\\ \hline V_{14}\\ \hline V_{20}\\ V_{22}\\ V_{23}\\ V_{24}\\ V_{25}\\ V_{26}\\ V_{27}\\ V_{28}\\ V_{29}\\ V_{30}\\ V_{31}\\ V_{32}\\ V_{33}
\end{array}\right] = 0 \qquad (2\text{-}2)
$$

$$\begin{bmatrix} 0 \\ V_{20} \\ V_{22} \\ V_{23} \\ V_{24} \\ V_{25} \\ V_{26} \\ V_{27} \\ V_{28} \\ V_{29} \\ V_{30} \\ V_{31} \\ V_{32} \\ V_{33} \end{bmatrix} = - \begin{bmatrix} \underline{B}_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \\ V_{12} \\ V_{13} \\ V_{14} \end{bmatrix} \qquad (2\text{-}3)$$

From the system, we have the terminal equations in the form.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \\ V_{12} \\ V_{13} \\ V_{14} \end{bmatrix} = \underbrace{\begin{bmatrix} Z_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{66} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{77} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{88} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{99} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{1010} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{1111} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{1212} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{1414} \end{bmatrix}}_{Z_N}$$

(2-4)

where Z_N consists of the values of line impedances of positive sequence. As we are studying balanced fault, we can neglect the mutual impedance as shown in equation (2-4).

Now transforming the brench currents in terms of chord currents from the fundamental cut-set equations we have

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \\ I_{10} \\ I_{11} \\ I_{12} \\ I_{13} \\ I_{14} \end{bmatrix} = \begin{bmatrix} \underline{B}_1^T \end{bmatrix} \begin{bmatrix} I_{14} \\ I_{20} \\ I_{22} \\ I_{23} \\ I_{24} \\ I_{25} \\ I_{26} \\ I_{27} \\ I_{28} \\ I_{29} \\ I_{30} \\ I_{31} \\ I_{32} \\ I_{33} \end{bmatrix} \qquad (2\text{-}5)$$

Then, substituting (2-4) and (2-5) into (2-3) we have:

$$\begin{bmatrix} 0 \\ V_{20} \\ V_{22} \\ V_{23} \\ V_{24} \\ V_{25} \\ V_{26} \\ V_{27} \\ V_{28} \\ V_{29} \\ V_{30} \\ V_{31} \\ V_{32} \\ V_{33} \end{bmatrix} = - \begin{bmatrix} \underline{B}_1 \end{bmatrix} \begin{bmatrix} \underline{Z}_N \end{bmatrix} \begin{bmatrix} B_1^T \end{bmatrix} \begin{bmatrix} I_{14} \\ I_{20} \\ I_{22} \\ I_{23} \\ I_{24} \\ I_{25} \\ I_{26} \\ I_{27} \\ I_{28} \\ I_{29} \\ I_{30} \\ I_{31} \\ I_{32} \\ I_{33} \end{bmatrix} \qquad (2-6)$$

From the top equation of equation (2-6) we can find the value of I_{14} and eliminate it from the rest of the equations of the equation (2-6). This gives us a relation between

$$\begin{bmatrix} V_{20} \\ V_{22} \\ V_{23} \\ V_{24} \\ V_{25} \\ V_{26} \\ V_{27} \\ V_{28} \\ V_{29} \\ V_{30} \\ V_{31} \\ V_{32} \\ V_{33} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} I_{20} \\ I_{22} \\ I_{23} \\ I_{24} \\ I_{25} \\ I_{26} \\ I_{27} \\ I_{28} \\ I_{29} \\ I_{30} \\ I_{31} \\ I_{32} \\ I_{33} \end{bmatrix}$$

From the graph of Figure 2-2 we can observe that:

$$\begin{bmatrix} V_{20} \\ V_{22} \\ V_{23} \\ V_{24} \\ V_{25} \\ V_{26} \\ V_{27} \\ V_{28} \\ V_{29} \\ V_{30} \\ V_{31} \\ V_{32} \\ V_{33} \end{bmatrix} = \begin{bmatrix} V_{T'_0} \\ V_{T'_2} \\ V_{T'_3} \\ V_{T'_4} \\ V_{T'_5} \\ V_{T'_6} \\ V_{T'_7} \\ V_{T'_8} \\ V_{T'_9} \\ V_{T'_{10}} \\ V_{T'_{11}} \\ V_{T'_{12}} \\ V_{T'_{13}} \end{bmatrix} \qquad \begin{bmatrix} I_{20} \\ I_{22} \\ I_{23} \\ I_{24} \\ I_{25} \\ I_{26} \\ I_{27} \\ I_{28} \\ I_{29} \\ I_{30} \\ I_{31} \\ I_{32} \\ I_{33} \end{bmatrix} = - \begin{bmatrix} I_{T'_0} \\ I_{T'_2} \\ I_{T'_3} \\ I_{T'_4} \\ I_{T'_5} \\ I_{T'_6} \\ I_{T'_7} \\ I_{T'_8} \\ I_{T'_9} \\ I_{T'_{10}} \\ I_{T'_{11}} \\ I_{T'_{12}} \\ I_{T'_{13}} \end{bmatrix}$$

So, we have now, one relationship between the voltages at the buses with respect to bus 1 and bus currents with respect to bus 1. Or:

$$\begin{bmatrix} V_{T'_0} \\ V_{T'_2} \\ V_{T'_3} \\ V_{T'_4} \\ V_{T'_5} \\ V_{T'_6} \\ V_{T'_7} \\ V_{T'_8} \\ V_{T'_9} \\ V_{T'_{10}} \\ V_{T'_{11}} \\ V_{T'_{12}} \\ V_{T'_{13}} \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \qquad\qquad \underline{Z}_{BN} \qquad\qquad \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} I_{T'_0} \\ I_{T'_2} \\ I_{T'_3} \\ I_{T'_4} \\ I_{T'_5} \\ I_{T'_6} \\ I_{T'_7} \\ I_{T'_8} \\ I_{T'_9} \\ I_{T'_{10}} \\ I_{T'_{11}} \\ I_{T'_{12}} \\ I_{T'_{13}} \end{bmatrix} \tag{2-7}$$

Our next step is to determine a terminal equation in impedance form corresponding to the terminal graph (Figure 2-6a) for the combined system (generator and transmission systems).

Considering now the generator system (see graph, Figure 2-5)

where $V_{0'g} = 1 + j0$ (specified)

and $V_{00'} = Z_g I_{00'}$ (Z_g is the subtransient reactance of the generator.)

From the overall graph of Figure 2-7 (which is the transmission terminal representation (Figure 2.3a) connected to the augmented current drivers (graph, Figure 2.6b) and generator (graph, Figure 2-5)). The fundamental circuit equations can be written as:

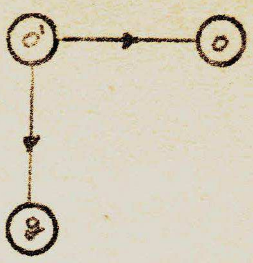


Figure 2-5

Terminal graph of the generator system.

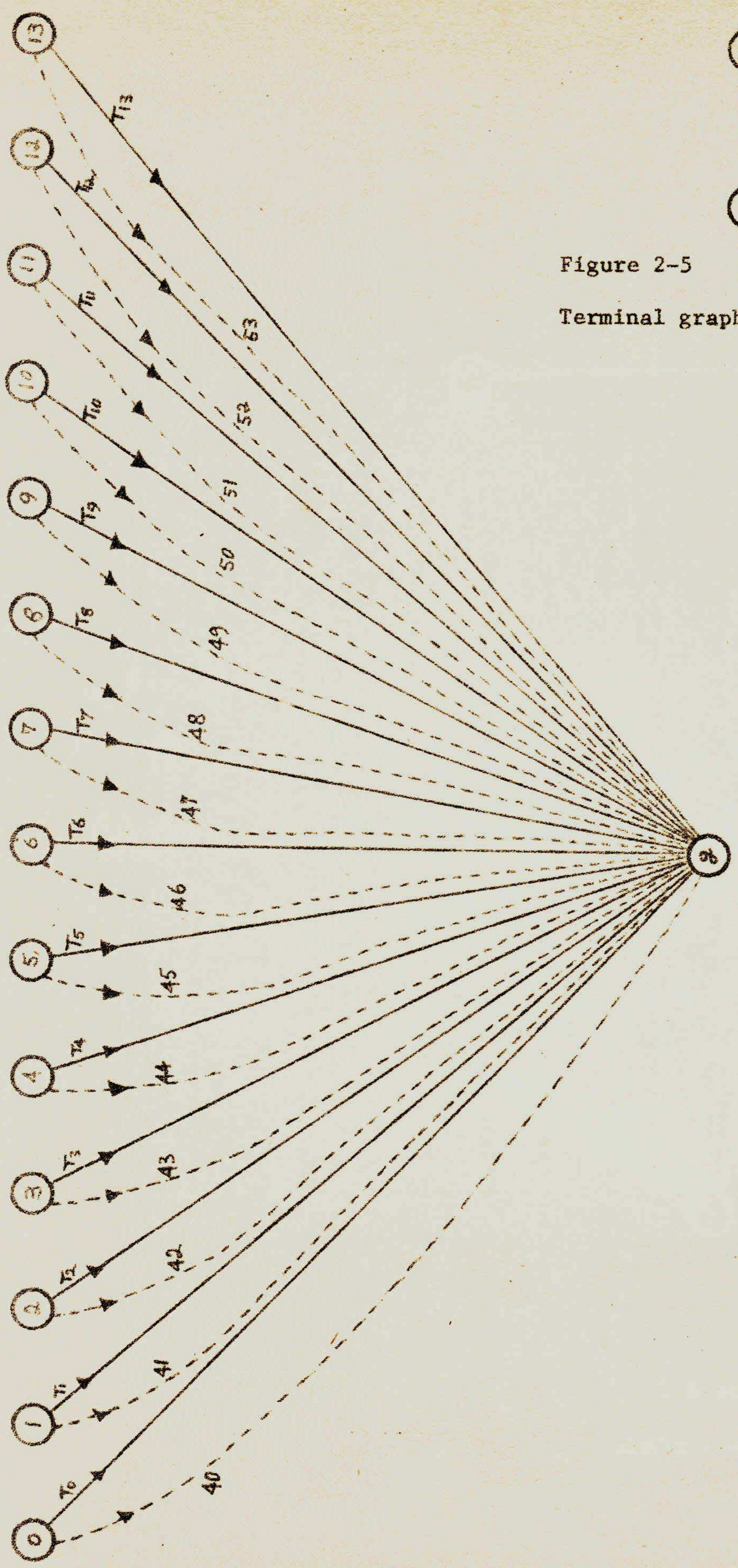


Figure 2-6

a) Solid lines - terminal graph of the combined system.

b) Dashed lines - augmented current drivers.

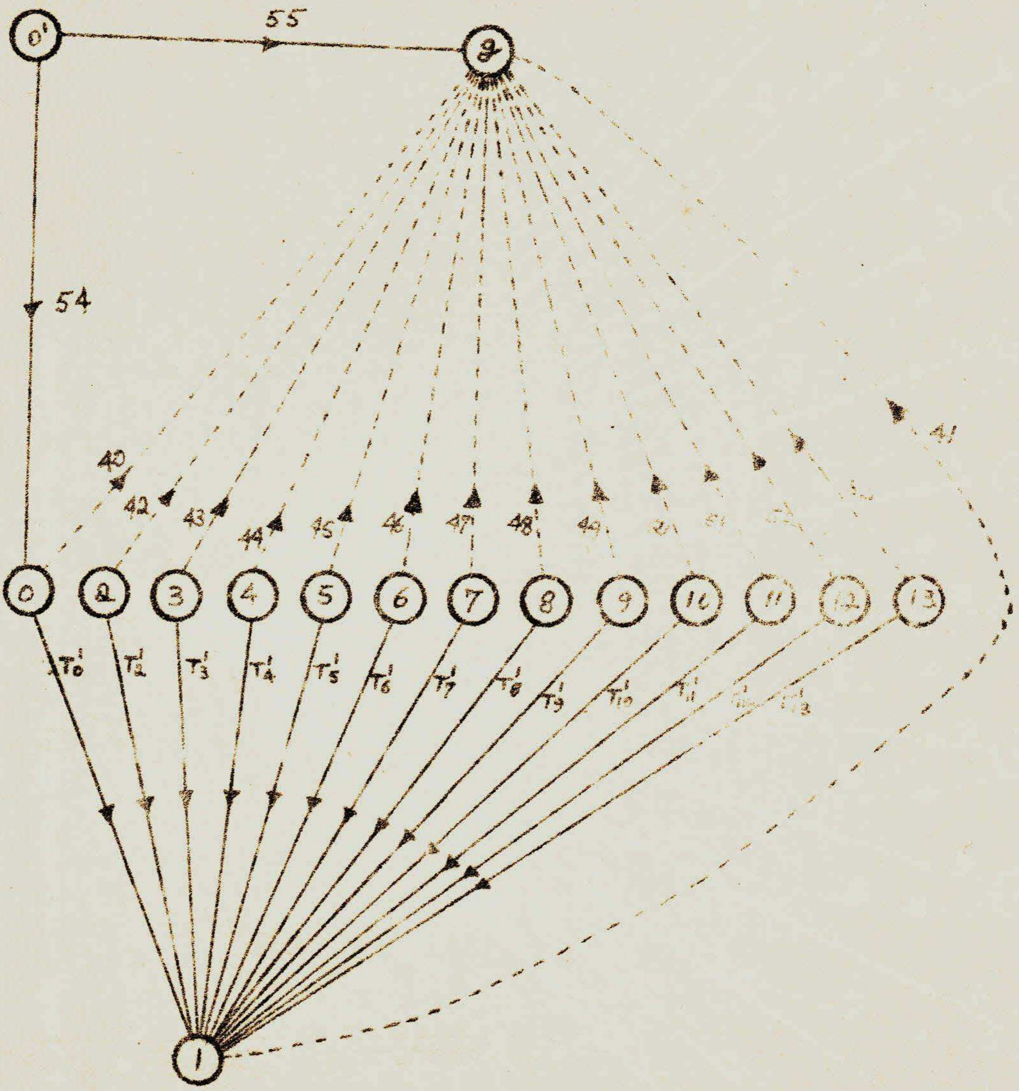


Figure 2-7

Overall graph.

$$\begin{array}{c} 40\\ 41\\ 42\\ 43\\ 44\\ 45\\ 46\\ 47\\ 48\\ 49\\ 50\\ 51\\ 52\\ 53 \end{array} \left[\underbrace{\begin{array}{cccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{array}}_{\underline{B}_2} \begin{array}{|c|} -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1\\ -1 \end{array} \begin{array}{cccccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array} \right] \left[\begin{array}{c}
V_{T',0}\\ V_{T',2}\\ V_{T',3}\\ V_{T',4}\\ V_{T',5}\\ V_{T',6}\\ V_{T',7}\\ V_{T',8}\\ V_{T',9}\\ V_{T',10}\\ V_{T',11}\\ V_{T',12}\\ V_{T',13}\\ V_{T',14}\\ V_{54}\\ V_{55}\\ \hline V_{40}\\ V_{41}\\ V_{42}\\ V_{43}\\ V_{44}\\ V_{45}\\ V_{46}\\ V_{47}\\ V_{48}\\ V_{49}\\ V_{50}\\ V_{51}\\ V_{52}\\ V_{53}
\end{array} \right] \qquad (2\text{-}8)$$

From equation (2-8) with $V_{55} = V_{0'g} = 1 + j0$ we have:

$$\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} (1 + j0) + \begin{bmatrix} \underline{B}_2 \end{bmatrix} \begin{bmatrix} V_{T'_0} \\ V_{T'_2} \\ V_{T'_3} \\ V_{T'_4} \\ V_{T'_5} \\ V_{T'_6} \\ V_{T'_7} \\ V_{T'_8} \\ V_{T'_9} \\ V_{T'_{10}} \\ V_{T'_{11}} \\ V_{T'_{12}} \\ V_{T'_{13}} \\ V_{54} \end{bmatrix} + \begin{bmatrix} V_{40} \\ V_{41} \\ V_{42} \\ V_{43} \\ V_{44} \\ V_{45} \\ V_{46} \\ V_{47} \\ V_{48} \\ V_{49} \\ V_{50} \\ V_{51} \\ V_{52} \\ V_{53} \end{bmatrix} = 0 \qquad (2\text{-}9)$$

We know also that $V_{54} = V_{00'} = Z_g\ I_{00'} = Z_g\ I_{54}$

$$V_{54} = Z_g\ I_{54} \qquad (2\text{-}10)$$

Substituting equation (2-7) and equation (2-10) into equation (2-9) we have:

$$\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} (1 + j0) + \begin{bmatrix} \underline{B}_2 \end{bmatrix} \left[\begin{array}{c|c} \underline{Z}_{BN} & \underline{0} \\ \hline \underline{0} & Z_g \end{array}\right] \begin{bmatrix} I_{T'_0} \\ I_{T'_2} \\ I_{T'_3} \\ I_{T'_4} \\ I_{T'_5} \\ I_{T'_6} \\ I_{T'_7} \\ I_{T'_8} \\ I_{T'_9} \\ I_{T'_{10}} \\ I_{T'_{11}} \\ I_{T'_{12}} \\ I_{T'_{13}} \\ I_{54} \end{bmatrix} + \begin{bmatrix} V_{40} \\ V_{41} \\ V_{42} \\ V_{43} \\ V_{44} \\ V_{45} \\ V_{46} \\ V_{47} \\ V_{48} \\ V_{49} \\ V_{50} \\ V_{51} \\ V_{52} \\ V_{53} \end{bmatrix} = 0 \qquad (2\text{-}11)$$

If we express the brench currents in terms of chord currents using the fundamental cut-set equations, the equation (2-11) becomes

$$\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} (1 + j0) + \begin{bmatrix} \underline{B}_2 \end{bmatrix} \left[\begin{array}{c|c} \underline{Z}_{BN} & \underline{0} \\ \hline \underline{0} & Z_g \end{array}\right] \begin{bmatrix} \underline{B}_2^T \end{bmatrix} \begin{bmatrix} I_{40} \\ I_{41} \\ I_{42} \\ I_{43} \\ I_{44} \\ I_{45} \\ I_{46} \\ I_{47} \\ I_{48} \\ I_{49} \\ I_{50} \\ I_{51} \\ I_{52} \\ I_{53} \end{bmatrix} + \begin{bmatrix} V_{40} \\ V_{41} \\ V_{42} \\ V_{43} \\ V_{44} \\ V_{45} \\ V_{46} \\ V_{47} \\ V_{48} \\ V_{49} \\ V_{50} \\ V_{51} \\ V_{52} \\ V_{53} \end{bmatrix} = 0 \qquad (2\text{-}12)$$

From the graph of Figure 2-6 we can observe that:

$$\begin{bmatrix} V_{40} \\ V_{41} \\ V_{42} \\ V_{43} \\ V_{44} \\ V_{45} \\ V_{46} \\ V_{47} \\ V_{48} \\ V_{49} \\ V_{50} \\ V_{51} \\ V_{52} \\ V_{53} \end{bmatrix} = \begin{bmatrix} V_{T_0} \\ V_{T_1} \\ V_{T_2} \\ V_{T_3} \\ V_{T_4} \\ V_{T_5} \\ V_{T_6} \\ V_{T_7} \\ V_{T_8} \\ V_{T_9} \\ V_{T_{10}} \\ V_{T_{11}} \\ V_{T_{12}} \\ V_{T_{13}} \end{bmatrix} = \underline{V}_{BUS} \qquad \begin{bmatrix} I_{40} \\ I_{41} \\ I_{42} \\ I_{43} \\ I_{44} \\ I_{45} \\ I_{46} \\ I_{47} \\ I_{48} \\ I_{49} \\ I_{50} \\ I_{51} \\ I_{52} \\ I_{53} \end{bmatrix} = - \begin{bmatrix} I_{T_0} \\ I_{T_1} \\ I_{T_2} \\ I_{T_3} \\ I_{T_4} \\ I_{T_5} \\ I_{T_6} \\ I_{T_7} \\ I_{T_8} \\ I_{T_9} \\ I_{T_{10}} \\ I_{T_{11}} \\ I_{T_{12}} \\ I_{T_{13}} \end{bmatrix} = - \underline{I}_{BUS}$$

So, the equation (2-12) becomes:

$$\begin{bmatrix} V_{T_0} \\ V_{T_1} \\ V_{T_2} \\ V_{T_3} \\ V_{T_4} \\ V_{T_5} \\ V_{T_6} \\ V_{T_7} \\ V_{T_8} \\ V_{T_9} \\ V_{T_{10}} \\ V_{T_{11}} \\ V_{T_{12}} \\ V_{T_{13}} \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{B}_2 \end{bmatrix} \begin{bmatrix} \underline{Z}_{BN} & \underline{0} \\ \underline{0} & Z_g \end{bmatrix} \begin{bmatrix} \underline{B}_2^T \end{bmatrix}}_{\underline{Z}_{BUS}} \begin{bmatrix} I_{T_0} \\ I_{T_1} \\ I_{T_2} \\ I_{T_3} \\ I_{T_4} \\ I_{T_5} \\ I_{T_6} \\ I_{T_7} \\ I_{T_8} \\ I_{T_9} \\ I_{T_{10}} \\ I_{T_{11}} \\ I_{T_{12}} \\ I_{T_{13}} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} (1 + j0)$$

$$\underline{V}_{BUS} = \underline{Z}_{BUS}\, \underline{I}_{BUS} + bV_o \qquad (2\text{-}13)$$

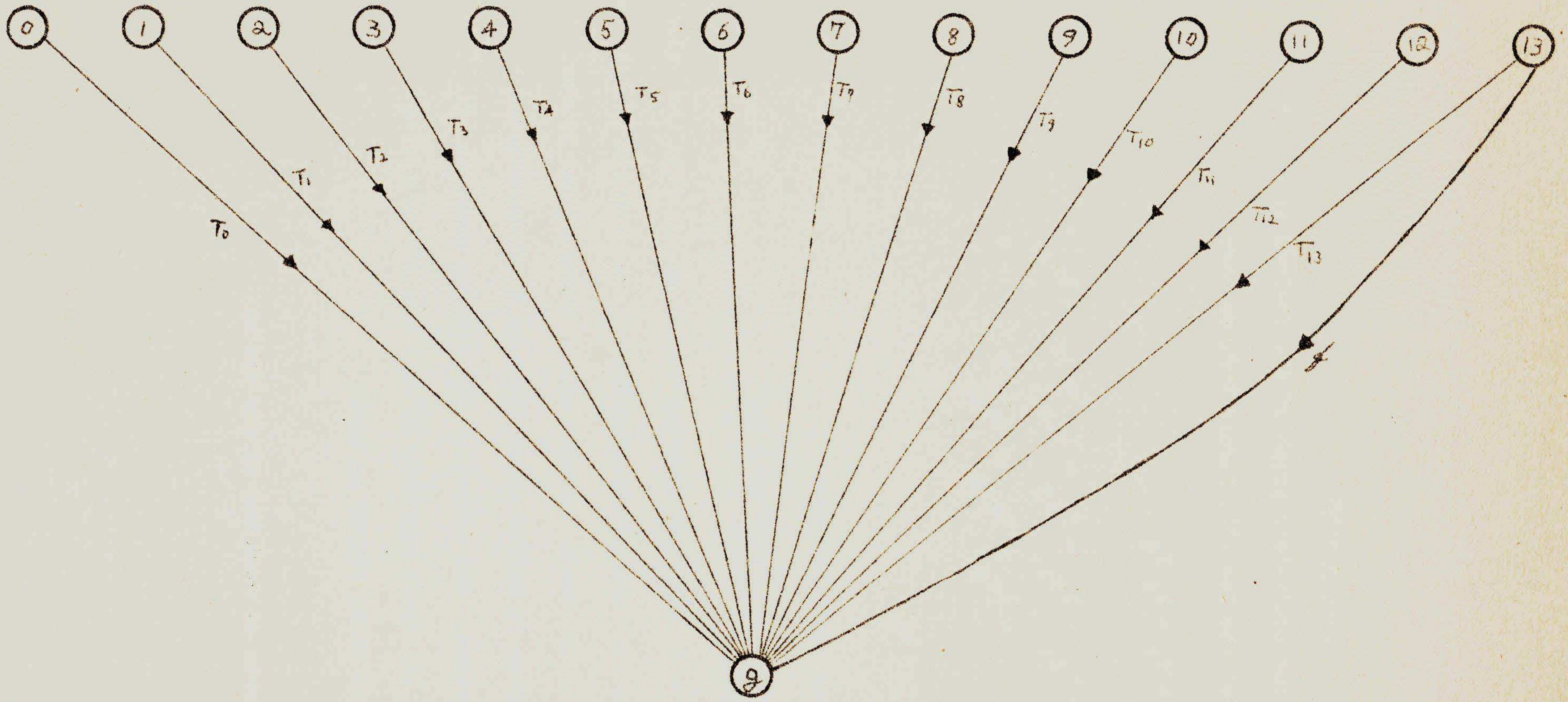


Figure 2-8

Equivalent graph of the faulted system.

SOLUTION OF THE FAULTED SYSTEM

From the equivalent graph of the faulted system (for example, fault at the bus 13), Figure 2-8, we have:

$$V_{T_{13}} = V_f = Z_f \, I_f$$

$$I_{T_0} = I_{T_1} = I_{T_2} = I_{T_3} = I_{T_4} = I_{T_5} = I_{T_6} = I_{T_7} = I_{T_8}$$
$$= I_{T_9} = I_{T_{10}} = I_{T_{11}} = I_{T_{12}} = 0$$

$$I_{T_{13}} = - I_f$$

From the equation (2-13) we have:

$$I_f = (Z_f + Z_{1313})^{-1} \, (1 + j0) \qquad (2\text{-}14)$$

If we assume $Z_f = 0$, from the equation (2-14) we can find the value of I_f at the bus 13. Using this procedure, from equation (2-13), we can determine all the fault currents at all buses of the system.

A program in WATFIV was-written by M. Chandrashekar applying this method. The output of the program consists of: i) Z_{BUS} matrix, ii) fault currents at each bus of the system.

For the CHESF SYSTEM we have:

Program output.

ii) fault currents at each bus of the system.

BUS	FAULT CURRENTS	
	MAGNITUDE	ANGLE
0 - Generator terminal	32.2581	-90.0
1 - PAULO AFONSO	20.8333	-90.0
2 - CATU	4.4840	-81.67
3 - ITABAIANA	5.7545	-82.37
4 - BOM NOME	4.6858	-81.38
5 - MILAGRES	3.3793	-80.66
6 - BANABUIU	1.9309	-79.99
7 - FORTALEZA	1.4516	-79.80
8 - MATATU	3.3070	-81.00
9 - ANGELIN	8.8778	-83.88
10 - CAMPINA GRANDE	1.5251	-85.89
11 - SANTA CRUZ	1.2380	-83.17
12 - RECIFE	6.0121	-82.75
13 - GOIANINHA	1.5016	-82.32

2.4 LINE TO GROUND FAULT

From the equation (2-14), $I_f = (Z_f + Z_{1313})^{-1}$ (1 + j0), for a fault at the bus p, whose fault impedance is Z_f

$$I_f = (Z_f + Z_{pp})^{-1} V_0 \qquad (2\text{-}15)$$

where Z_{pp} is element of Z_{BUS}.

For unbalanced faults we can express equation (2-15) in terms of symmetrical components.

$$I_F^{0,1,2} = (Z_F^{0,1,2} + Z_{pp}^{0,1,2})^{-1} V_0^{0,1,2} \qquad (2\text{-}16)$$

or

$$I_F^{0,1,2} = Y_F^{0,1,2} (\underline{U} + Z_{pp}^{0,1,2} Y_F^{0,1,2})^{-1} V_0^{0,1,2} \qquad (2\text{-}17)$$

where:

$$V_0^{0,1,2} = \frac{1}{\sqrt{3}} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \underbrace{\begin{vmatrix} 1 \\ 1\angle -120^\circ \\ 1\angle 120^\circ \end{vmatrix}}_{V_0^{a,b,c}} = \begin{vmatrix} 0 \\ \sqrt{3} \\ 0 \end{vmatrix}$$

and

$$Y_F^{0,1,2} = 1/Z_F^{0,1,2}$$

Terminal representation for three phase to ground fault

The Figure 2-9 illustrates the terminal representation for three phase to ground fault, and we can represent all types of fault using this representation.

The terminal graph, as shown in Figure 2-10a, is obtained using the chord formulation to graph Figure 2-11, which has the terminal equation

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_g \end{bmatrix} \begin{bmatrix} z_a & 0 & 0 & 0 \\ 0 & z_b & 0 & 0 \\ 0 & 0 & z_c & 0 \\ 0 & 0 & 0 & z_g \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_g \end{bmatrix}$$

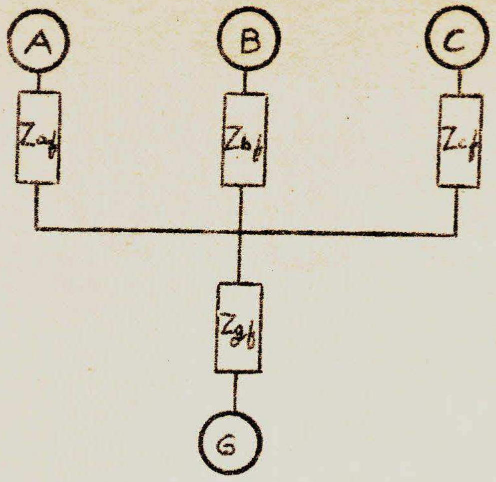


Figure 2-9

Terminal representation for three phase to ground fault.

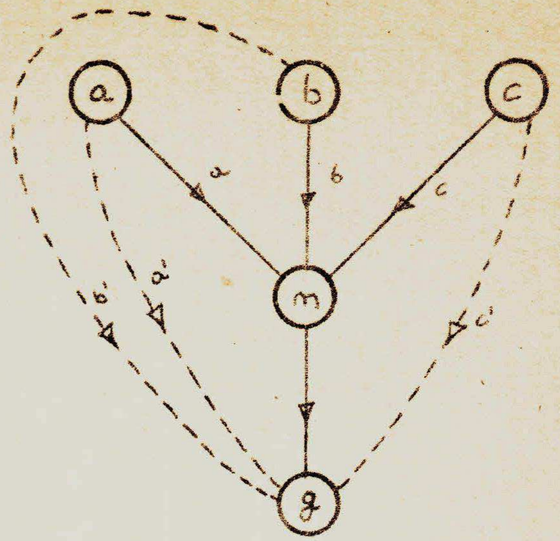


Figure 2-11

Augmented graph.

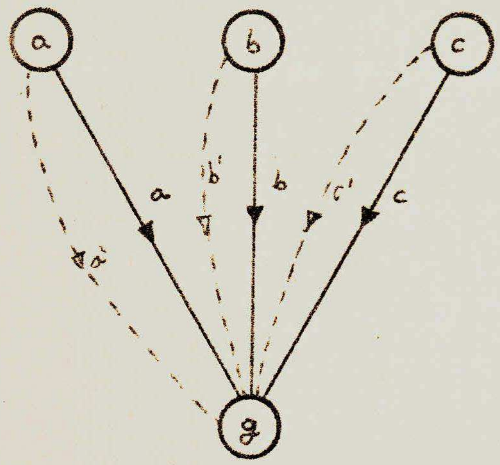


Figure 2-10

a) Solid lines - terminal graph

b) Dashed lines - augmented source drivers.

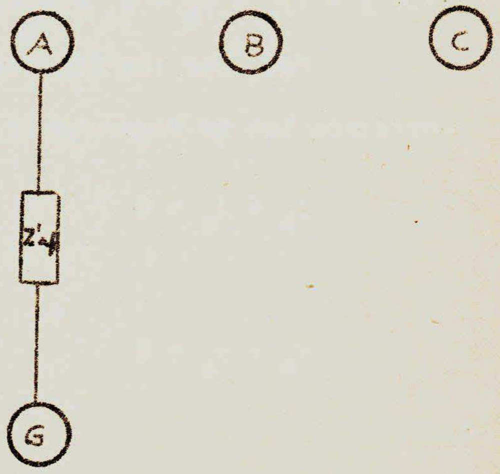


Figure 2-12

Terminal representation for line to ground fault.

We can get the $Z_F^{a,b,c}$ matrix as follows:

$$\begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} = \underbrace{\begin{bmatrix} (z_a + z_g) & z_g & z_g \\ z_g & (z_b + z_g) & z_g \\ z_g & z_g & (z_c + z_g) \end{bmatrix}}_{Z_F^{a,b,c}} \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}$$

or

$$\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix} = \frac{1}{y} \overbrace{\begin{bmatrix} y_a(y_b + y_c + y_g) & -y_a y_b & -y_a y_c \\ -y_a y_b & y_b(y_a + y_b + y_g) & -y_b y_c \\ -y_a y_c & -y_b y_c & y_c(y_a + y_b + y_g) \end{bmatrix}}^{Y_F^{a,b,c}} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}$$

where $y = (y_a + y_b + y_c + y_g)$.

Now, for the line to ground fault, we can see the terminal representation in Figure 2-12, i.e.

$$I_b = I_c = 0$$

and

$$y_b = y_c = 0$$

then:

$$Y_F^{a,b,c} = \begin{bmatrix} \frac{y_a y_g}{y_a + y_g} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with

$$\frac{y_a + y_g}{y_a + y_g} = y_f$$

we have:

$$Y_F^{0,1,2} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} y_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} = \frac{y_f}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Equation (2-17) can be written as:

$$\text{(2-17)} \qquad I_F^{0,1,2} = Y_F^{0,1,2} \, [U + Z_{pp}^{0,1,2} \, Y_F^{0,1,2}]^{-1} \, V_0^{0,1,2}$$

$$I_F^{0,1,2} = \frac{y_f}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} Z_{pp}^{(0)} & & \\ & Z_{pp}^{(1)} & \\ & & Z_{pp}^{(2)} \end{bmatrix} \frac{y_f}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right]^{-1} \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix}$$

at $Z_f = 1/y_f$ and $Z_{pp}^{(1)} = Z_{pp}^{(2)}$.

$$I_F^{0,1,2} = \frac{\sqrt{3}}{Z_{pp}^{(0)} + 2Z_{pp}^{(1)} + 3Z_f} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$I_F^{a,b,c} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} I_F^{0,1,2} = \begin{vmatrix} \dfrac{3}{Z_{44}^{(0)} + 2Z_{44}^{(1)} + 3Z_f} \\ 0 \\ 0 \end{vmatrix}$$

The bus voltages may be determined from equation (2-13) in terms of symmetrical components at fault condition.

$$V_F^{0,1,2} = - Z_{pp}^{0,1,2} \; I_F^{0,1,2} + \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix}$$

then:

$$V_F^{a,b,c} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} V_F^{0,1,2}$$

so,

$$\begin{vmatrix} \dfrac{3Z_f}{Z_{pp}^{(0)} + 2Z_{pp}^{(1)} + 3Z_f} \\ \\ a^2 - \dfrac{Z_{pp}^{(0)} - Z_{pp}^{(1)}}{Z_{pp}^{(0)} + 2Z_{pp}^{(1)} + 3Z_f} \\ \\ a - \dfrac{Z_{pp}^{(0)} - Z_{pp}^{(1)}}{Z_{pp}^{(0)} + 2Z_{pp}^{(1)} +3Z_f} \end{vmatrix}$$

for the CHESF SYSTEM, we can determine $Z_{pp}^{0,1,2}$ at all buses of the system using the program written by M. Chandrashekar.

BUS (p)	$Z_{pp}^{(0)}$	$Z_{pp}^{(1)}$
0 - Generator terminal	j0.0310	j0.0310
1 - PAULO AFONSO	j0.0480	j0.0480
2 - CATU	0.1244 + j0.4982	0.0323 + j0.2207
3 - ITABAIANA	0.0902 + j0.3723	0.0231 + j0.1722
4 - BOM NOME	0.1340 + j0.4940	0.0320 + j0.2110
5 - MILAGRES	0.2020 + j0.7170	0.0480 + j0.2920
6 - BANABUIU	0.3680 + j1.2970	0.0900 + j0.5100
7 - FORTALEZA	0.4980 + j1.7450	0.1220 + j0.6780
8 - MATATU	0.1822 + j0.7012	0.0473 + j0.2987
9 - ANGELIN	0.0700 + j0.3060	0.0120 + j0.1120
10 - CAMPINA GRANDE	0.2150 + j1.1570	0.0470 + j0.6540
11 - SANTA CRUZ	0.3950 + j1.7270	0.0960 + j0.8020
12 - RECIFE	0.1310 + j0.5100	0.0210 + j0.1650
13 - GOIANINHA	0.2620 + j1.4260	0.0890 + j0.6600

Now, assuming $Z_f = 0$ and using the following program in WATFIV, we can determine $V_F^{a,b,c}$ and $I_F^{a,b,c}$ at all buses of the system for line to ground fault.

```
$JOB    WATFIV
        COMPLEX A1, A2, ZO, Z1, CFA, VFB, VFC, CMPLX
        A = - 0.5
        B = 0.866
        M = 14
        DO 10 I = 1, M
        READ, ROX XO, R1, X1
        A1 = CMPLX (A, B)
        A2 = CMPLX (A, -B)
        ZO = CMPLX (RO, XO)
        Z1 = CMPLX (R1, X1)
        CFA = 3.0/(ZO + 2.0 * Z1)
        VFB = A2 - (ZO - Z1)/(ZO + 2.0 * Z1)
        VFC = A1 - (ZO - Z1)/(ZO + 2.0 * Z1)
        PRINT, 'BUS NO', I, 'IFA = ', CFA, 'VFB = ', VFB, VFB, 'VFC = ', VFC
  10    CONTINUE
        STOP
        END
$ENTRY
```

Output of the program

$I_F^b = I_F^c = V_F^a = 0$ for all buses of the system

BUS	I_F^a	V_F^b	V_F^c
0 - Generator terminal	- j32.258	- 0.500 - j0.866	- 0.500 + j0.866
1 - PAULO AFONSO	- j20.833	- 0.500 - j0.866	- 0.500 + j0.866
2 - CATU	0.617 - j3.069	- 0.803 - j0.829	- 0.803 + j0.903
3 - ITABAIANA	0.769 - j4.040	- 0.787 - j0.827	- 0.787 + j0.905
4 - BOM NOME	0.676 - j3.129	- 0.818 - j0.823	- 0.818 + j0.908
5 - MILAGRES	0.502 - j2.191	- 0.836 - j0.825	- 0.836 + j0.907
6 - BANABUIU	0.290 - j1.226	- 0.848 - j0.828	- 0.848 + j0.903
7 - FORTALEZA	0.219 - j0.915	- 0.853 - j0.829	- 0.853 + j0.903
8 - MATATU	0.470 - j2.210	- 0.818 - j0.829	- 0.818 + j0.902
9 - ANGELIN	0.973 - j5.488	- 0.874 - j0.823	- 0.874 + j0.909
10 - CAMPINA GRANDE	0.150 - j1.198	- 0.709 - j0.824	- 0.709 + j0.908
11 - SANTA CRUZ	0.154 - j0.874	- 0.785 - j0.824	- 0.785 + j0.906
12 - RECIFE	0.706 - j3.426	- 0.920 - j0.822	- 0.920 + j0.910
13 - GOIANINHA	0.171 - j1.065	- 0.782 - j0.848	- 0.782 + j0.884

3. LOAD FLOW SIMULATION

3.1 CONSIDERATIONS

For load flow problem we will assume that the system is balanced. Therefore we will take into account only the positive sequence components. We will not neglect the shunt paths (line charging, static capacitor or static reactor). The reactances of the transformers will be neglected and transformers with off nominal taps will be correctly represented.

For load flow studies, the buses will be classified into the following types:

1) Voltage controlled buses (generator buses) - At these buses P and $|V|$ are specified and, Q and $\underline{|V}$ are not specified.
2) Load buses - At these buses, net P and net Q are specified, $|V|$ and $\underline{|V}$ are not specified.
3) Slack-bus - At this bus, P and Q are not specified, V and $\underline{|V}$ may or may not be specified, depending on numerical scheme.

3.2 TERMINAL EQUATIONS

From the above considerations, we can derive Z_{BUS} or Y_{BUS} for the terminal equation which describes the power system performance, with the ground as reference.

$$I_{BUS} = Y_{BUS} V_{BUS} \qquad (3-1)$$

or

$$V_{BUS} = Z_{BUS}\ I_{BUS} \quad . \tag{3-2}$$

Now, we will show how we can obtain the Y_{BUS} matrix for the system. We have to derive terminal equation in the admittance form corresponding to the terminal graph, Figure 3-2a. From Figure 3-3, the augmented graph (union of the system graph, Figure 3-1, with the augmented source drivers, Figure 3-2b), we will write the fundamental cut set equations, choosing the graph of the augmented source drivers as tree. Then, the fundamental cut-set equations will be of the form,

$$[\underline{u} \,\vdots\, \underline{A}] \begin{bmatrix} \underline{I}_A \\ \hdashline \underline{I}_N \end{bmatrix} = 0 \tag{3-3}$$

where $\underline{I}_A$ corresponds to the elements of the augmented source drivers, and $\underline{I}_N$ corresponds to the elements of the system graph. $[u \,\vdots\, \underline{A}]$ is the fundamental cut set matrix.

From equation (3-3) we have:

$$\underline{I}_A = - \underline{A}\ \underline{I}_N \tag{3-4}$$

and from the system we have the terminal equations

$$\underline{I}_N = \underline{Y}_N\ \underline{V}_N \tag{3-5}$$

From circuit equations, the chord voltages can be expressed in terms of brench voltages as:

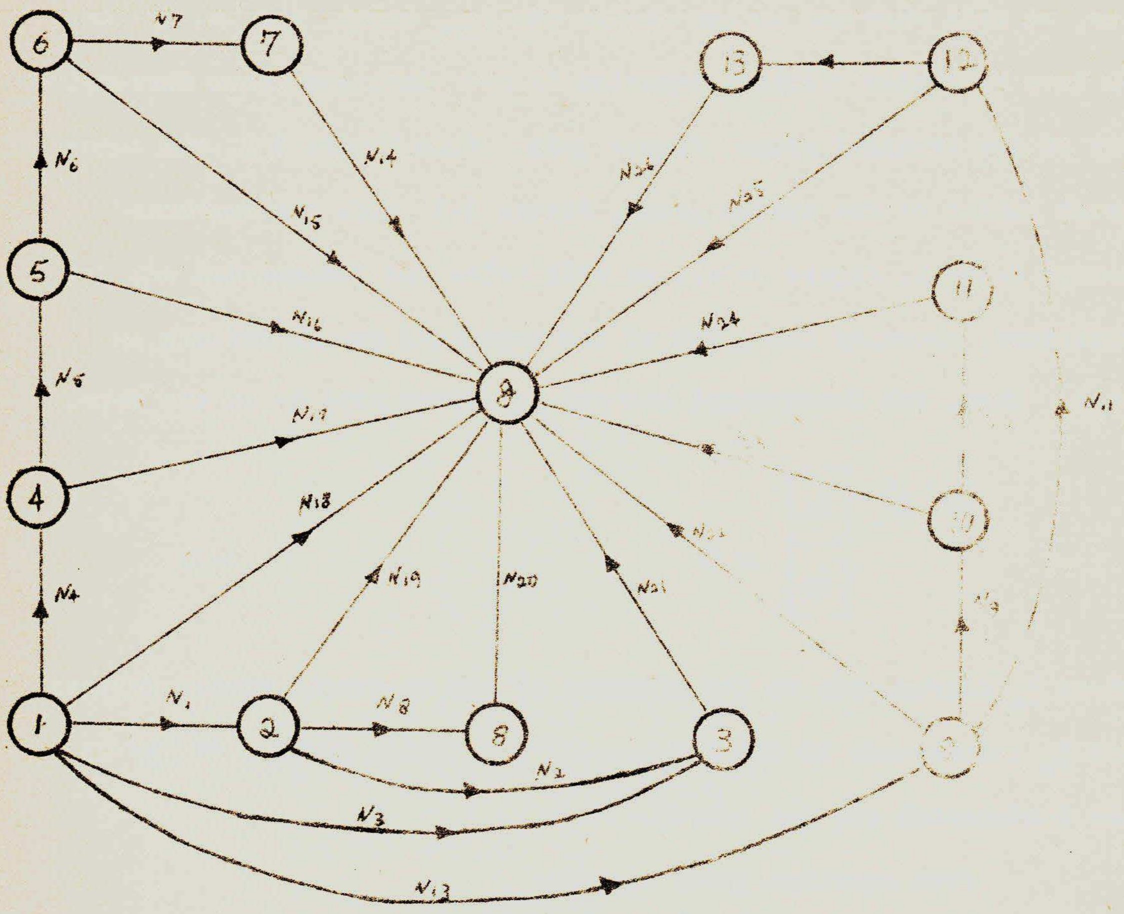


Figure 3-1

Graph of the CHESF system.

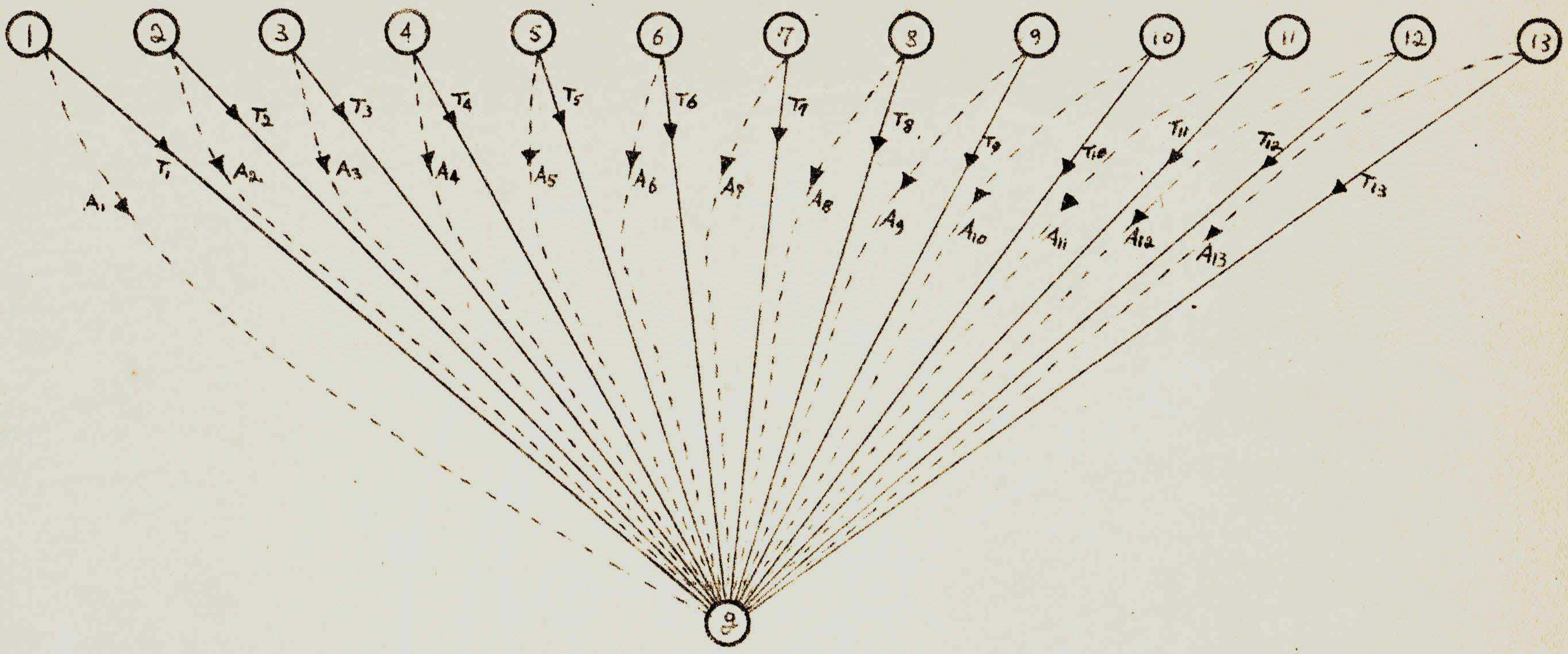


Figure 3-2

a) Solid lines - terminal graph.

b) Dashed lines - augmented source drivers.

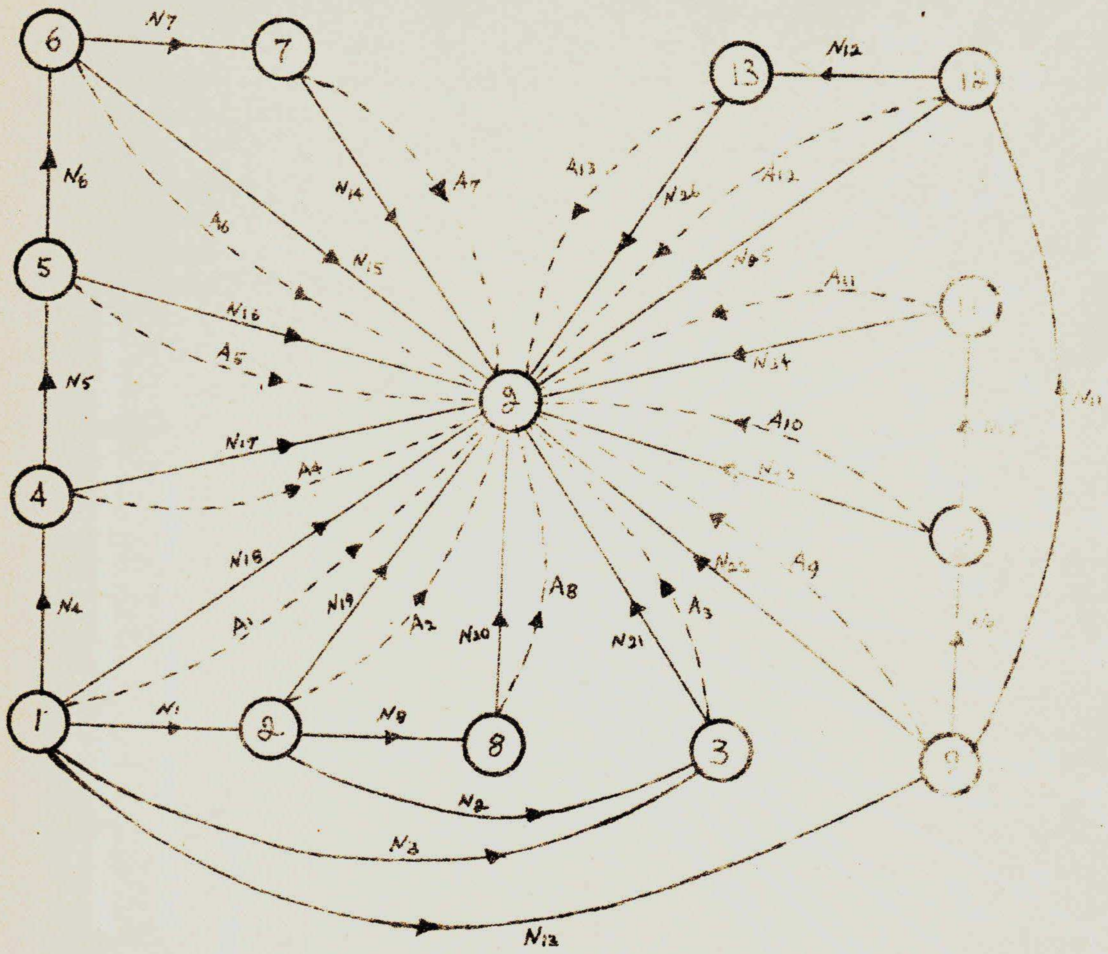


Figure 3-3

Augmented system graph.

$$\underline{V}_N = \underline{A}^T \, \underline{V_A} \tag{3-6}$$

substituting equation (3-5) and equation (3-6) into equation (3-4) we have:

$$\underline{I}_A = - A \, Y_N \, A^T \, \underline{V_A} \tag{3-7}$$

but, from the graph of Figure 3-2 we can see that:

$$- I_A = I_T = I_{BUS} \tag{3-8}$$

and

$$V_A = V_T = V_{BUS} \tag{3-9}$$

then, substituting equation (3-8) and equation (3-9) into equation (3-7) we have:

$$I_{BUS} = A \, Y_N \, A^T \, V_{BUS} \tag{3-10}$$

and

$$Y_{BUS} = A \, Y_N \, A^T \tag{3-11}$$

The Z_{BUS} matrix can be determined as in the case of short circuit formulation.

3.3 MATHEMATICAL FORMULATIONS FOR LOAD FLOW PROBLEM USING $\underline{Y}_{BUS}$ REPRESENTATION

Equation (3-1) $I_{BUS} = Y_{BUS} \, V_{BUS}$

or:

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q \qquad p = 1, 2 \ldots n \qquad (3\text{-}12)$$

where n is the number of buses.

Assuming that:

$$V_p^* I_p = P_p - j Q_p \qquad (3\text{-}13)$$

is the real and reactive power at the bus "p", we can calculate the current at all buses except at the slack bus in the form

$$I_p = \frac{P_p - j Q_p}{V_p^*} \qquad (3\text{-}14)$$

Substituting equation (3-14) into equation (3-12) we have:

$$P_p - j Q_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q \qquad (3\text{-}15)$$

$p \neq s$ where s is the slack bus.

These set of non linear equations can be solved using several numerical methods; assume that the system operates within "steady state stability limit".

Line flow equation

We know that:

$$I_{pq} = (V_p - V_q)\, y_{pq} + V_p\, y'_{pq}/2 \qquad (3\text{-}16)$$

where y_{pq} corresonds to the line admittance

and y'_{pq} corresponds to the total line charging admittance.

Also,

$$P_{pq} - j\, Q_{pq} = V_p^* \, I_{pq} \qquad (3\text{-}17)$$

Substituting equation (3-16) into equation (3-17) we have:

$$P_{pq} - j\, Q_{pq} = V_p^* \, (V_p - V_q) \, y_{pq} + V_p^* \, V_p \, y'_{pq}/2 \qquad (3\text{-}18)$$

Similarly, from the bus q to bus p is:

$$P_{qp} - j\, Q_{qp} = V_q^* \, (V_q - V_p) \, y_{pq} + V_q^* \, V_q \, y'_{pq}/2 \qquad (3\text{-}19)$$

Line-loss

The algebraic sum of the equations (3-18) and (3-19) give us the power loss in the line from the bus p to bus q.

3.4 LOAD FLOW SIMULATION OF THE CHESF SYSTEM WITH AVERAGE LOAD USING DIFFERENT METHODS

a) Gauss Seidel interative method using Y_{BUS} and voltage at the stack bus specified as 1.0.

BUS NAME	BUS VOLTAGE				BUS POWER	
	REAL	IMAGINARY	MAGNITUDE	ANGLE	REAL	REACTIVE
P.AFONSO	1.00000	0.00000	1.00000	0.00000	3.37383	-3.17607
CATU	0.99162	-0.16431	1.00514	-9.40829	-0.46810	-0.22800
ITABAIA.	1.01384	-0.09128	1.01794	-5.14447	-0.16150	-0.07870
B.NOME	1.14444	-0.10255	1.14903	-5.12056	-0.02410	-0.01170
MILAGRES	1.19798	-0.14644	1.20687	-6.95956	-0.07040	-0.03430
BANABUIU	1.28633	-0.23210	1.30710	-10.22831	-0.00820	-0.00400
FORTALE.	1.28638	-0.27060	1.31454	-11.87927	-0.27300	-0.13290
MATATU	0.97184	-0.18825	0.98991	-10.96281	-0.36860	-0.17950
ANGELIN	1.03066	-0.13197	1.03907	-7.29654	-0.22950	-0.11180
C.GRANDE	1.05655	-0.18102	1.07194	-9.72236	-0.08230	-0.04010
S.CRUZ	1.04975	-0.19967	1.06857	-10.76947	-0.13430	-0.06540
RECIFE	0.99566	-0.19719	1.01500	-11.20254	-1.11430	-0.55650
GOIANIN.	0.94977	-0.22689	0.97649	-13.43605	-0.26590	-0.12950

	REAL	REACTIVE
TRANSMISSION NETWORK LOSS	0.17311	-4.75004

```
 *****LINE FLOWS*****

  BUS   TO BUS          REAL POWER   REACTIVE POWER

FONSO  CATU              0.500772       -0.349697
FONSO  ITABAIA.          0.529455       -0.315478
FONSO  B.NOME            0.438294       -1.116230
FONSO  ANGELIN           1.905190       -1.394265
U      P.AFONSO         -0.485450       -0.135494
U      ITABAIA.         -0.353525       -0.158544
U      MATATU            0.370984        0.065977
AIA.   P.AFONSO         -0.520036        0.096528
AIA.   CATU              0.358488       -0.175213
OME    P.AFONSO         -0.401899        0.967496
OME    MILAGRES          0.377361       -0.979849
AGRES  B.NOME           -0.366120        0.839603
AGRES  BANABUIU          0.295725       -0.873954
ABUIU  MILAGRES         -0.282818        0.345919
ABUIU  FORTALE.          0.274489       -0.350228
TALE.  BANABUIU         -0.272868       -0.132681
ATU    CATU             -0.368691       -0.179438
LIN    P.AFONSO         -1.853239        0.510870
LIN    C.GRANDE          0.220678       -0.401547
LIN    RECIFE            1.403055       -0.221113
ANDE   ANGELIN          -0.217371        0.066135
ANDE   S.CRUZ            0.135137       -0.106350
UZ     C.GRANDE         -0.134343       -0.065360
IFE    ANGELIN          -1.385994       -0.657403
IFE    GOIANIN.          0.271608        0.100803
ANIN.  RECIFE           -0.265925       -0.129480
```

b) Gauss Seidel interative method using Y_{BUS} when the voltage at the slack bus is not specified.

```
NAME                      BUS VOLTAGE                          BUS POWER
        REAL     IMAGINARY   MAGNITUDE     ANGLE        REAL      REACTIVE

NSO   0.96214    0.13087    0.97100     7.74578     3.37229   -2.78588
      0.96496   -0.03769    0.96569    -2:23702    -0.46810   -0.22800
IA.   0.98195    0.03986    0.98276     2.32478    -0.16150   -0.07870
ME    1.11062    0.04736    1.11165     2.46748    -0.02410   -0.01170
GRES  1.16601    0.01117    1.16606     0.54870    -0.07040   -0.03430
BUIU  1.25832   -0.06276    1.25989    -2.85542    -0.00820   -0.00400
ALE.  1.26105   -0.10179    1.26515    -4.61461    -0.27300   -0.13290
TU    0.94714   -0.06489    0.94936    -3.91918    -0.36860   -0.17950
LIN   1.00002    0.00089    1.00002     0.05106    -0.22950   -0.11180
ANDE  1.02841   -0.04527    1.02940    -2.52035    -0.08230   -0.04010
OZ    1.02279   -0.06503    1.02485    -3.63821    -0.13430   -0.06540
FE    0.96969   -0.07061    0.97225    -4.16468    -1.11430   -0.55650
NIA.  0.92526   -0.10706    0.93143    -6.60048    -0.26590   -0.12950
```

```
                                                    REAL          ACTIVE

        TRANSMISSION NETWORK LOSS                 0.17210       -4.35828
```

```
                   *****LINE FLOWS*****

FROM BUS  TO BUS          REAL POWER  REACTIVE POWER

P.AFONSO  CATU              0.501567      -0.299487
P.AFONSO  ITABAIA.          0.530133      -0.267722
P.AFONSO  B.NOME            0.433505      -1.030689
P.AFONSO  ANGELIN           1.907463      -1.187427
CATU      P.AFONSO         -0.435580      -0.142574
CATU      ITABAIA.         -0.353775      -0.160946
CATU      MATATU            0.371173       0.077182
ITABAIA.  P.AFONSO         -0.520546       0.065642
ITABAIA.  CATU              0.359144      -0.144580
B.NOME    P.AFONSO         -0.399944       0.887920
B.NOME    MILAGRES          0.375449      -0.899676
MILAGRES  B.NOME           -0.365038       0.767847
MILAGRES  BANABUIU          0.294812      -0.802387
BANABUIU  MILAGRES         -0.282893       0.310212
BANABUIU  FORTALE.          0.274636      -0.314284
FORTALE.  BANABUIU         -0.272962      -0.132799
MATATU    CATU             -0.368660      -0.179646
ANGELIN   P.AFONSO         -1.855588       0.379972
ANGELIN   C.GRANDE          0.220631      -0.361793
ANGELIN   RECIFE            1.405414      -0.129102
C.GRANDE  ANGELIN          -0.217474       0.052326
C.GRANDE  S.CRUZ            0.135175      -0.092289
S.CRUZ    C.GRANDE         -0.134323      -0.065499
RECIFE    ANGELIN          -1.386629      -0.662808
RECIFE    GOIANIN.          0.272420       0.106283
GOIANIN.  RECIFE           -0.265920      -0.129628
```

The Figure 3-4 illustrates the effect of acceleration factors on the rate of convergence for load flow solutions for both cases, where the optimum acceleration factor is 1.5 corresponding to 15 interactions and accuracy of 0.0001.

Execution time for 15 interactions in IBM 3601, Model 75

slack bus voltage	specified	3.47s
slack bus voltage	not specified	3.60s

This simulation has been done using a program in WATFIV written by M. V. Bhat.

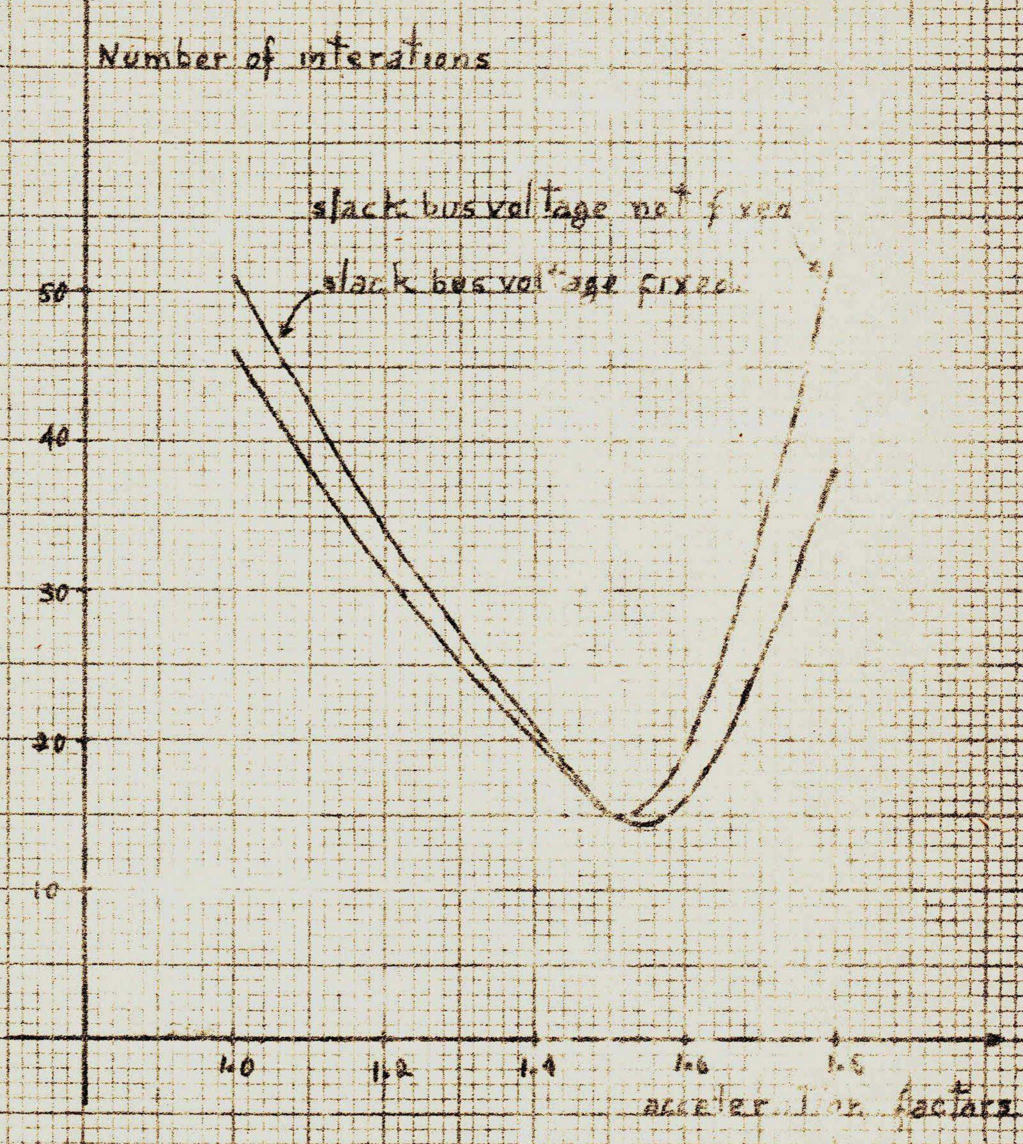

Number of interations
slack bus voltage not fixed
slack bus voltage fixed
50
40
30
20
10
1.0
1.2
1.4
1.6
1.8
acceleration factors

Figure 3-4

c) Newton Raphson method using Y_{BUS} with the voltage at the slack bus specified as 1.0.

The program was developed by Dr. K. Genesan at Memorial University of Newfoundland and modified to suit the University of Waterloo computing facilities.

```
                               LINE FLOWS
FROM BUS   TO BUS       REAL POWER   REACTIVEPOWER
P.AFONSO  CATU           0.500768     -0.349722
P.AFONSO  ITABAIA.       0.529424     -0.316489
P.AFONSO  B.NOME         0.437936     -1.117278
P.AFONSO  ANGELIN        1.905307     -1.394437
CATU      P.AFONSO      -0.485447     -0.135474
CATU      ITABAIA.      -0.353542     -0.158516
CATU      MATATU         0.370893      0.066035
ITABAIA.  P.AFONSO      -0.520005      0.096535
ITABAIA.  CATU           0.358505     -0.175242
B.NOME    P.AFONSO      -0.401486      0.968764
B.NOMF    MILAGRES       0.377386     -0.980412
MILAGRES  B.NOME        -0.366137      0.840142
MILAGRES  BANABUIU       0.295740     -0.874415
BANABUIU  MILAGRES      -0.282825      0.346204
BANABUIU  FORTALE.       0.274622     -0.350196
FORTALE.  BANABUIU      -0.273000     -0.132896
MATATU    CATU          -0.368601     -0.179502
ANGELIN   P.AFONSO      -1.853349      0.511075
ANGELIN   C.GRANDF       0.220704     -0.401632
ANGELIN   RECIFE         1.403141     -0.221219
C.GRANDF  ANGELIN       -0.217395      0.066217
C.GRANDE  S.CRUZ         0.135095     -0.106317
S.CRUZ    C.GRANDE      -0.134301     -0.065402
RECIFE    ANGELIN       -1.386078     -0.657314
RECIFE    GOIANIN.       0.271782      0.100819
GOIANIN.  RECIFE        -0.265900     -0.129499
```

REAL LINE LOSS 0.1732

NUMBER OF INTERATIONS

EXECUTION TIME 9.95 se

IN IBM 3601 MODEL 75

```
BUS        VOLT MAG   ANGLE DEG   REAL POWER  REACTIVE POWER
P.AFONSO  1.00000    -  0.0000    3.3734     -3.1779
CATU      1.00515      -9.4082   -0.4681     -0.2280
ITABAIA.  1.01794      -5.1442   -0.1615     -0.0787
B.NOME    1.14921      -5.1185   -0.0241     -0.0117
MILAGRES  1.20708      -6.9673   -0.0704     -0.0343
BANABUIU  1.30735     -10.2256   -0.0082     -0.0040
FORTALE.  1.31477     -11.[illegible]  -0.2730     -0.1329
MATATU    0.98991     -10.9623   -0.3686     -0.1795
ANGELIN   1.03908      -7.2970   -0.2295     -0.1118
C.GRANDF  1.07197      -9.7231   -0.0823     -0.0401
S.CRUZ    1.06859     -10.7697   -0.1343     -0.0654
RECIFE    1.01502     -11.2032   -1.1143     -0.5565
GOIANIN.  0.97651     -13.4363   -0.2659     -0.1295
```

3.5 LOAD FLOW STUDY OF THE CHESF SYSTEM USING Y_{BUS} REPRESENTATION AND GAUSS SEIDEL INTERATIVE METHOD WITH ACCELERATION FACTOR 1.5

3.5.1 PRESENT SITUATION OF THE SYSTEM

Initially, we will do one analysis of the present situation of the system, having PAULO AFONSO as slack bus, where all generators are situated, and assuming voltage of magnitude 1.06 for it.

We will assume the following cases for simulation of the system.

a) Maximum load with all reactors on, except in the load at MILAGRES.

b) Maximum load with all reactors off.

c) Average load with all reactors on.

d) Average load with all reactors off.

And for the set of lines from PAULO AFONSO to FORTALEZA (cases e and f) below,

e) Maximum load with reactor on, only in the load at MILAGRES.

f) Average load with all reactors on, except in the load at MILAGRES.

OBSERVATION

All the static shunt capacitors in the load will be introduced only in the cases of maximum load.

Case a) Maximum load with all reactors on, except in the load at MILAGRES (Bus

```
BUS NAME                        BUS VOLTAGE                             BUS POWER
            REAL      IMAGINARY  MAGNITUDE      ANGLE          REAL       REACTI

P.AFONSO   1.06000    0.00000    1.06000     0.00000       7.25696     1.095
CATU       0.80949   -0.27236    0.85424   -18.62795      -0.86550    -0.431
ITABAIA.   0.95859   -0.15743    0.97143    -9.32649      -0.36150    -0.087
B.NOME     0.98539   -0.15405    0.99736    -8.83557      -0.05780    -0.028
MILAGRES   0.93536   -0.22082    0.96109   -13.28301      -0.16500    -0.039
BANABUIU   0.80323   -0.35950    0.88018   -24.13545      -0.02350     0.006
FORTALE.   0.67463   -0.44227    0.80701   -33.28355      -0.70330    -0.126
MATATU     0.74384   -0.31535    0.80739   -22.96786      -0.73060    -0.355
ANGELIN    0.95492   -0.23420    0.98322   -13.78021      -0.53000    -0.156
C.GRANDE   0.93061   -0.31541    0.98261   -18.72311      -0.15860    -0.023
S.CRUZ     0.90277   -0.35220    0.96904   -21.31248      -0.28650    -0.011
RECIFE     0.84020   -0.35664    0.91276   -23.00003      -2.22350    -0.690
GOIANIN.   0.72389   -0.40734    0.83062   -29.36673      -0.54600    -0.167
```

```
                                                      REAL          REACTIVE

              TRANSMISSION NETWORK LOSS             0.58633         -1.05573
```

```
         *****LINE FLOWS *****

FROM BUS  TO BUS          REAL POWER  REACTIVE POWER

P.AFONSO  CATU              1.028472        0.321055
P.AFONSO  ITABAIA.          1.114750        0.307673
P.AFONSO  B.NOME            1.056360        0.115993
P.AFONSO  ANGELIN           4.062466        0.351677
CATU      P.AFONSO         -0.945807       -0.429476
CATU      ITABAIA.         -0.684213       -0.418704
CATU      MATATU            0.744519        0.343908
ITABAIA.  P.AFONSO         -1.075992       -0.373328
ITABAIA.  CATU              0.714493        0.285424
B.NOME    P.AFONSO         -1.022383       -0.247955
B.NOME    MILAGRES          0.964847        0.220422
MILAGRES  B.NOME           -0.948511       -0.273926
MILAGRES  BANABUIU          0.783592        0.101436
BANABUIU  MILAGRES         -0.752232       -0.257954
BANABUIU  FORTALE.          0.729014        0.187466
FORTALE.  BANABUIU         -0.703387       -0.256841
MATATU    CATU             -0.730000       -0.355503
ANGELIN   P.AFONSO         -3.875976       -0.523484
ANGELIN   C.GRANDE          0.457118       -0.219450
ANGELIN   RECIFE            2.888861        0.586833
C.GRANDE  ANGELIN          -0.449392       -0.046773
C.GRANDE  S.CRUZ            0.290792       -0.072984
S.CRUZ    C.GRANDE         -0.286501       -0.053802
RECIFE    ANGELIN          -2.801184       -0.905656
RECIFE    GOIANIN.          0.577682        0.214994
GOIANIN.  RECIFE           -0.546000       -0.167900
```

Case b) Maximum load with all reactors off.

```
 BUS NAME  |                      BUS VOLTAGE                   |         BUS POWER
           |     REAL    IMAGINARY  MAGNITUDE        ANGLE      |      REAL    | REACT
           |                                                    |              |
P.AFONSO   |   1.06000    0.00000    1.06000       0.00000     |    7.21747   | -0.3
CATU       |   0.83911   -0.27654    0.88351     -18.23996     |   -0.86550   | -0.4
ITABAIA.   |   0.97206   -0.15871    0.98493      -9.27324     |   -0.36150   | -0.0
B.NOME     |   1.13086   -0.18032    1.14514      -9.05962     |   -0.05760   | -0.0
MILAGRES   |   1.15019   -0.25975    1.17915     -12.72586     |   -0.16500   | -0.0
BANABUIU   |   1.16734   -0.42352    1.24179     -19.94135     |   -0.02350   |  0.0
FORTALE.   |   1.12119   [illegible] 1.23194     -24.43057     |   -0.70330   | -0.1
MATATU     |   0.77667   -0.31826    0.83935     -22.28226     |   -0.73000   | -0.3
ANGELIN    |   0.97276   -0.23684    1.00118     -13.68399     |   -0.53000   | -0.1
C.GRANDE   |   0.97633   -0.32955    1.03045     -18.65172     |   -0.15860   | -0.0
S.CRUZ     |   0.95665   -0.36976    1.02562     -21.13228     |   -0.28650   | -0.0
RECIFE     |   0.86291   -0.35805    0.93424     -22.53510     |   -2.22350   | -0.6
GOIANIN.   |   0.75070   -0.40900    0.85489     -28.58233     |   -0.54600   | -0.1
```

	REAL	REAC
TRANSMISSION NETWORK LOSS	0.54728	-2.4

```
*****LINE FLOWS *****

FROM BUS  TO BUS        REAL POWER  REACTIVE POWER

P.AFONSO  CATU           1.017446       0.224062
P.AFONSO  ITABAIA.       1.107096       0.220958
P.AFONSO  B.NOME         1.041930      -0.827155
P.AFONSO  ANGELIN        4.051249       0.058375
CATU      P.AFONSO      -0.946543      -0.382890
CATU      ITABAIA.      -0.682322      -0.330023
CATU      MATATU         0.743439       0.331676
ITABAIA.  P.AFONSO      -1.070707      -0.302923
ITABAIA.  CATU           0.709196       0.215009
B.NOME    P.AFONSO      -0.998461       0.698252
B.NOME    MILAGRES       0.941022      -0.725705
MILAGRES  B.NOME        -0.925336       0.613296
MILAGRES  BANABUIU       0.760392      -0.693495
BANABUIU  MILAGRES      -0.737286       0.262123
BANABUIU  FORTALE.       0.714084      -0.255139
FORTALE.  BANABUIU      -0.703477      -0.126716
MATATU    CATU          -0.730031      -0.355514
ANGELIN   P.AFONSO      -3.870945      -0.283044
ANGELIN   C.GRANDE       0.458386      -0.392726
ANGELIN   RECIFE         2.882710       0.519968
C.GRANDE  ANGELIN       -0.449131       0.113386
C.GRANDE  S.CRUZ         0.290558      -0.136566
S.CRUZ    C.GRANDE      -0.286527      -0.011875
RECIFE    ANGELIN       -2.799383      -0.899336
RECIFE    GOIANIN.       0.575914       0.208653
GOIANIN.  RECIFE        -0.546028      -0.167900
```

Case c) Average load with all reactors on.

```
BUS NAME                     BUS VOLTAGE                          BUS POWER
              REAL    IMAGINARY  MAGNITUDE     ANGLE         REAL     REACTIVE

P.AFONSO     1.06000   0.00000    1.06000     0.00000      3.31164   -2.53393
CATU         1.05083  -0.15409    1.06206    -8.34219     -0.46810   -0.22800
ITABAIA.     1.07603  -0.08645    1.07949    -4.59344     -0.16150   -0.07870
B.NOME       1.05645  -0.06139    1.05823    -3.32580     -0.02410   -0.01170
MILAGRES     1.03776  -0.08562    1.04129    -4.71641     -0.07040   -0.33430
BANABUIU     1.02513  -0.14584    1.03545    -8.09590     -0.00820   -0.00400
FORTALE.     0.97782  -0.17796    0.99388   -10.31461     -0.27300   -0.13290
MATATU       1.03317  -0.17729    1.04827    -9.73723     -0.36660   -0.17950
ANGELIN      1.09646  -0.12527    1.10361    -6.52297     -0.22950   -0.11180
C.GRANDE     1.09816  -0.16349    1.11026    -8.46763     -0.08230   -0.04010
S.CRUZ       1.08506  -0.17813    1.09958    -9.32236     -0.13430   -0.06540
RECIFE       1.06863  -0.18830    1.08509    -9.99352     -1.11430   -0.55650
GOIANIN.     1.02715  -0.21735    1.04990   -11.94781     -0.26590   -0.12950
```

	REAL	REACTIVE
TRANSMISSION NETWORK LOSS	0.11223	-4.40593

```
*****LINE FLOWS *****

FROM BUS  TO BUS         REAL POWER   REACTIVE POWER

P.AFONSO CATU             0.498591      -0.379951
P.AFONSO ITABAIA.         0.528138      -0.352101
P.AFONSO B.NOME           0.388786      -0.215017
P.AFONSO ANGELIN          1.696558      -1.586473
CATU     P.AFONSO        -0.485104      -0.183062
CATU     ITABAIA.        -0.353607      -0.207510
CATU     MATATU           0.370716       0.049573
ITABAIA. P.AFONSO        -0.519594       0.093814
ITABAIA. CATU             0.358044      -0.172481
B.NOME   P.AFONSO        -0.384401      -0.085701
B.NOME   MILAGRES         0.360582       0.074569
MILAGRES B.NOME          -0.358336      -0.219941
MILAGRES BANABUIU         0.287986      -0.222819
BANABUIU MILAGRES        -0.284760      -0.165844
BANABUIU FORTALE.         0.276631       0.054790
FORTALE. BANABUIU        -0.273103      -0.330580
MATATU   CATU            -0.368695      -0.179453
ANGELIN  P.AFONSO        -1.848310       0.537210
ANGELIN  C.GRANDE         0.218936      -0.273223
ANGELIN  RECIFE           1.399911      -0.375867
C.GRANDE ANGELIN         -0.217371      -0.106060
C.GRANDE S.CRUZ           0.135132      -0.057406
S.CRUZ   C.GRANDE        -0.134354      -0.125815
RECIFE   ANGELIN         -1.385165      -0.648746
RECIFE   GOIANIN.         0.270974       0.092158
GOIANIN. RECIFE          -0.265928      -0.129475
```

Case d) Average load with all reactors off.

```
BUS NAME                    BUS VOLTAGE                            BUS POWER
               REAL    IMAGINARY  MAGNITUDE      ANGLE           REAL     REACTIV

P.AFONSO     1.06000    0.00000    1.06000     0.00000        3.38091   -3.9935
CATU         1.07323   -0.15361    1.08489    -8.40697       -0.46810   -0.2280
ITABAIA.     1.08606   -0.08838    1.08965    -4.65229       -0.16150   -0.0787
B.NOME       1.22146   -0.10336    1.22535    -4.83688       -0.02410   -0.0117
MILAGRES     1.28200   -0.14747    1.29050    -6.55173       -0.07040   -0.0343
BANABUIU     1.38304   -0.23325    1.40346    -9.56696       -0.00620   -0.0040
FORTALE.     1.38893   -0.27077    1.41508   -11.03144       -0.27300   -0.1329
MATATU       1.05617   -0.18139    1.07164    -9.74523       -0.36860   -0.1795
ANGELIN      1.11061   -0.12847    1.11801    -6.59862       -0.22950   -0.111
C.GRANDE     1.14410   -0.17667    1.15765    -8.77811       -0.08230   -0.040
S.CRUZ       1.13983   -0.19495    1.15645    -9.70511       -0.13430   -0.065
RECIFE       1.08376   -0.19080    1.10066    -9.98265       -1.11430   -0.556
GOIANIN.     1.04329   -0.21053    1.06614   -11.33258       -0.26590   -0.129
```

```
                                                            REAL           REACTIVE

          TRANSMISSION NETWORK LOSS                      0.17998          -5.56814
```

```
*****LINE FLOWS *****


FROM BUS  TO BUS          REAL POWER  REACTIVE POWER

P.AFONSO CATU               0.499529       -0.454364
P.AFONSO ITABAIA.           0.528584       -0.417848
P.AFONSO B.NOME             0.448719       -1.300029
P.AFONSO ANGELIN            1.903774       -1.822101
CATU     P.AFONSO          -0.485289       -0.117922
CATU     ITABAIA.          -0.353299       -0.153145
CATU     MATATU             0.370633        0.042966
ITABAIA. P.AFONSO          -0.519201        0.161164
ITABAIA. CATU               0.357640       -0.239359
B.NOME   P.AFONSO          -0.406087        1.138998
B.NOME   MILAGRES           0.381451       -1.151649
MILAGRES B.NOME            -0.368275        0.993416
MILAGRES BANABUIU           0.297889       -1.027714
BANABUIU MILAGRES          -0.282772        0.422781
BANABUIU FORTALE.           0.274407       -0.427174
FORTALE. BANABUIU          -0.272840       -0.132614
MATATU   CATU              -0.368707       -0.179453
ANGELIN  P.AFONSO          -1.849811        0.785458
ANGELIN  C.GRANDE           0.220978       -0.486295
ANGELIN  RECIFE             1.399283       -0.411004
C.GRANDE ANGELIN           -0.217275        0.095782
C.GRANDE S.CRUZ             0.135050       -0.135975
S.CRUZ   C.GRANDE          -0.134341       -0.065376
RECIFE   ANGELIN           -1.384979       -0.646854
RECIFE   GOIANIN.           0.270807        0.090246
GOIANIN. RECIFE            -0.265923       -0.129480
```

Case e) Maximum load with reactor only in the load at MILAGRES (Bus 5)

```
 BUS NAME                           BUS VOLTAGE                             BUS POWER
                REAL    IMAGINARY   MAGNITUDE      ANGLE          REAL      REA

P.AFONSO      1.06000    0.00000     1.06000     0.00000       7.20658     0.
CATU          0.83909   -0.27654     0.88349   -18.24065      -0.88550    -0.
ITABAIA.      0.97205   -0.15872     0.98492    -9.27339      -0.36150    -0.
B.NOME        1.05093   -0.16282     1.06352    -8.80634      -0.05780    -0.
MILAGRES      1.03220   -0.23332     1.05835   -12.75330      -0.16500    -0.
BANABUIU      1.01376   -0.40088     1.09016   -21.57521      -0.02350     0.
FORTALE.      0.94875   -0.49290     1.06916   -27.45261      -0.70330    -0.
MATAIU        0.77665   -0.31826     0.83933   -22.28296      -0.73000    -0.
ANGELIN       0.97272   -0.23685     1.00114   -13.68482      -0.53000    -0.
C.GRANDE      0.97629   -0.32956     1.03041   -18.65297      -0.15860    -0.
S.CRUZ        0.95661   -0.36977     1.02558   -21.13355      -0.28650    -0.
RECIFE        0.86246   -0.35805     0.93420   -22.58571      -2.22350    -0.
GOIANIN.      0.75065   -0.40800     0.85434   -28.53424      -0.54600    -0.
```

```
                                                  REAL          REACTI

           TRANSMISSION NETWORK LOSS            0.53587        -2.253
```

```
          *****LINE FLOWS *****

FROM BUS  TO BUS          REAL POWER  REACTIVE POWER

P.AFONSO CATU             1.017471      0.224131
P.AFONSO ITABAIA.         1.107113      0.221014
P.AFONSO B.NOME           1.030621     -0.305472
P.AFONSO ANGELIN          4.051452      0.059016
CATU     P.AFONSO        -0.946562     -0.382913
CATU     ITABAIA.        -0.682345     -0.380045
CATU     MATATU           0.743409      0.331670
ITABAIA. P.AFONSO        -1.070723     -0.302966
ITABAIA. CATU             0.709223      0.215059
B.NOME   P.AFONSO        -0.949783      0.137884
B.NOME   MILAGRES         0.942226     -0.165433
MILAGRES B.NOME          -0.929565      0.069697
MILAGRES BANABUIU         0.754623     -0.449858
BANABUIU MILAGRES        -0.740554      0.140780
BANABUIU FORTALE.         0.717312     -0.133828
FORTALE. BANABUIU        -0.703423     -0.126663
MATATU   CATU            -0.730001     -0.355505
ANGELIN  P.AFONSO        -3.871121     -0.283499
ANGELIN  C.GRANDE         0.458395     -0.392686
ANGELIN  RECIFE           2.882726      0.520065
C.GRANDE ANGELIN         -0.449139      0.113375
C.GRANDE S.CRUZ           0.290539     -0.135578
S.CRUZ   C.GRANDE        -0.286500     -0.011851
RECIFE   ANGELIN         -2.799390     -0.899312
RECIFE   GOIANIN.         0.575887      0.208659
GOIANIN. RECIFE          -0.546001     -0.167901
```

Case f) Average load with all reactors on, except in the load at MILAGRES (Bus 5)

```
BUS NAME                         BUS VOLTAGE                            BUS POWER
              REAL     IMAGINARY   MAGNITUDE      ANGLE            REAL      REACTIVE

P.AFUNSO    1.06000     0.00000    1.06000      0.00000         3.31749    -2.90336
CATU        1.05081    -0.15403    1.06203     -8.33907        -0.44810    -0.22800
ITABAIA.    1.07602    -0.08642    1.07948     -4.59204        -0.16150    -0.07870
B.NOME      1.11309    -0.07319    1.11550     -3.76179        -0.02410    -0.01170
MILAGRES    1.12157    -0.10300    1.12629     -5.24717        -0.07040    -0.03430
BANAPUIU    1.11692    -0.16058    1.12840     -8.18159        -0.00820    -0.00400
FORTALE.    1.07224    -0.18949    1.08885    -10.02181        -0.27300    -0.13290
MATATU      1.03315    -0.17723    1.04824     -9.73380        -0.36860    -0.17950
ANGELIN     1.09647    -0.12541    1.10362     -6.52506        -0.22950    -0.11180
C.GRANDE    1.09817    -0.16350    1.11027     -8.45814        -0.08230    -0.04010
S.CRUZ      1.08506    -0.17810    1.09958     -9.32125        -0.13430    -0.06540
RECIFE      1.06863    -0.18833    1.08509     -9.99481        -1.11430    -0.55650
GOIANIN.    1.02715    -0.21737    1.04990    -11.94910        -0.26590    -0.12950
```

	REAL	REACTIVE
TRANSMISSION NETWORK LOSS	0.11659	-4.47509

```
*****LINE FLOWS *****

FROM BUS  TO BUS          REAL POWER  REACTIVE POWER

P.AFONSO CATU              0.496406     -0.379547
P.AFONSO ITABAIA.          0.527975     -0.352027
P.AFONSO B.NOME            0.393006     -0.584214
P.AFONSO ANGELIN           1.997203     -1.585674
CATU     P.AFONSO         -0.484930     -0.183204
CATU     ITABAIA.         -0.353460     -0.207540
CATU     MATATU            0.370633      0.049757
ITABAIA. P.AFONSO         -0.519436      0.093713
ITABAIA. CATU              0.357893     -0.172355
B.NOME   P.AFONSO         -0.383526      0.291524
B.NOME   MILAGRES          0.359694     -0.302857
MILAGRES B.NOME           -0.357438      0.135861
MILAGRES BANABUIU          0.287156     -0.297086
BANABUIU MILAGRES         -0.284214     -0.165017
BANABUIU FORTALE.          0.276156      0.034756
FORTALE. BANABUIU         -0.273058     -0.370113
MATATU   CATU             -0.368512     -0.179531
ANGELIN  P.AFONSO         -1.845925      0.537560
ANGELIN  C.GRANDE          0.218748     -0.273231
ANGELIN  RECIFE            1.399636     -0.375718
C.GRANDE ANGELIN          -0.217136     -0.105073
C.GRANDE S.CRUZ            0.134900     -0.057193
S.CRUZ   C.GRANDE         -0.134124     -0.126038
RECIFE   ANGELIN          -1.384895     -0.648940
RECIFE   GOIANIN.          0.270972      0.092149
GOIANIN. RECIFE           -0.265927     -0.129467
```

Now, we will plot the voltages of each bus versus Bus Number, Figures 3-5, 3-6, 3-7, 3-8 and 3-9. In this way we have a better visualization of the level of voltages. This will facilitate the study of the system regulation.

Figure 1 - Transmission lines set from PAULO AFONSO to RECIFE, buses 1, 9, 12 and 13.

Figure 2 - Transmission lines set from PAULO AFONSO to SANTA CRUZ, buses 1, 9, 10 and 11.

Figure 3 - Transmission lines set from PAULO AFONSO to FORTALEZA, buses 1, 4, 5, 6 and 7.

Figure 4 - Transmission lines set from PAULO AFONSO to PAULO AFONSO by the loop of buses 1, 2, 3 and 1.

Figure 5 - Transmission lines set from PAULO AFONSO to MACATU, busines 1, 2 and 8.

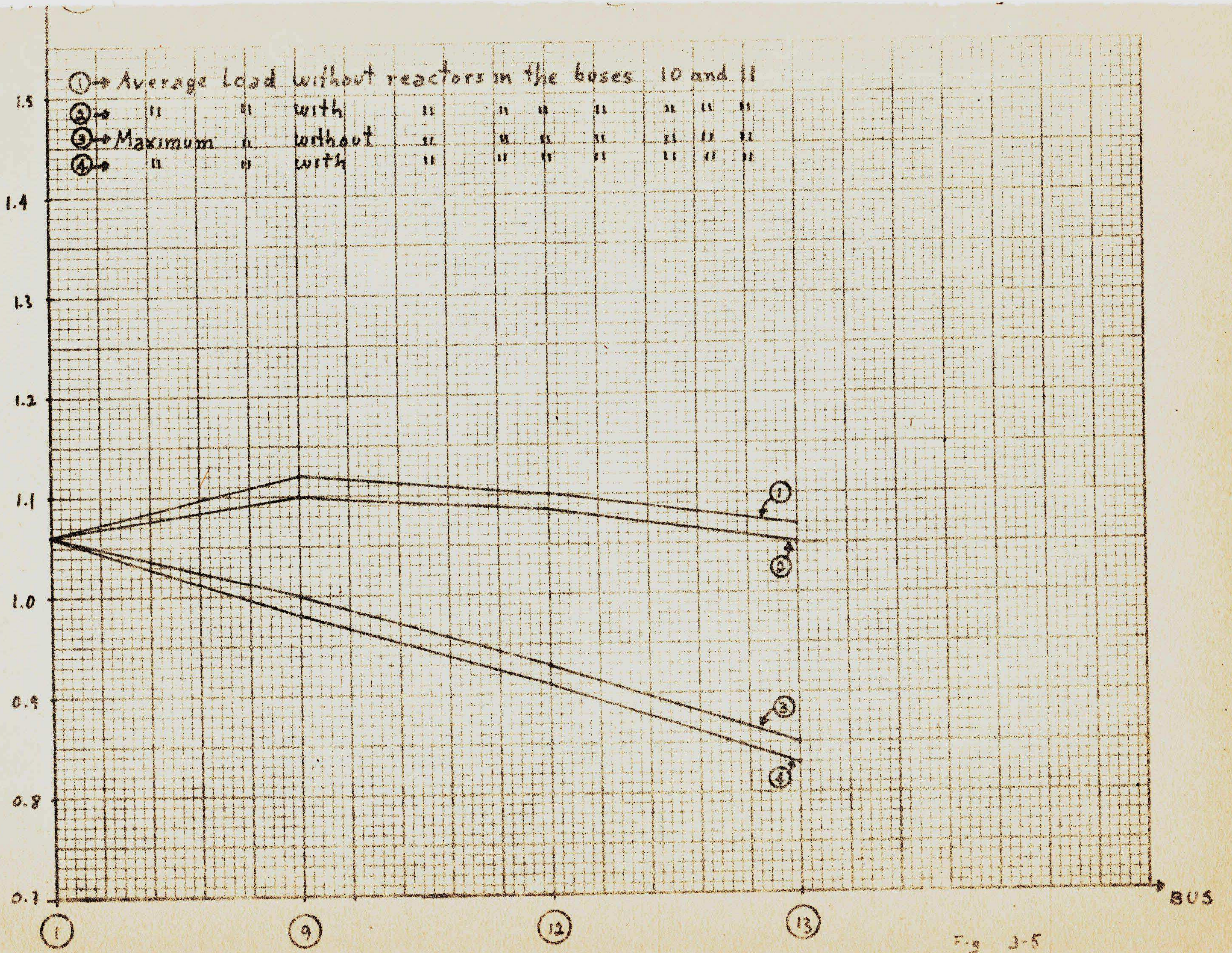

① → Average Load without reactors in the buses 10 and 11
② → " " with " " " " " " "
③ → Maximum " without " " " " " " "
④ → " " with " " " " " " "
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
①
②
③
④
BUS
①
⑨
⑫
⑬

Fig 3-5

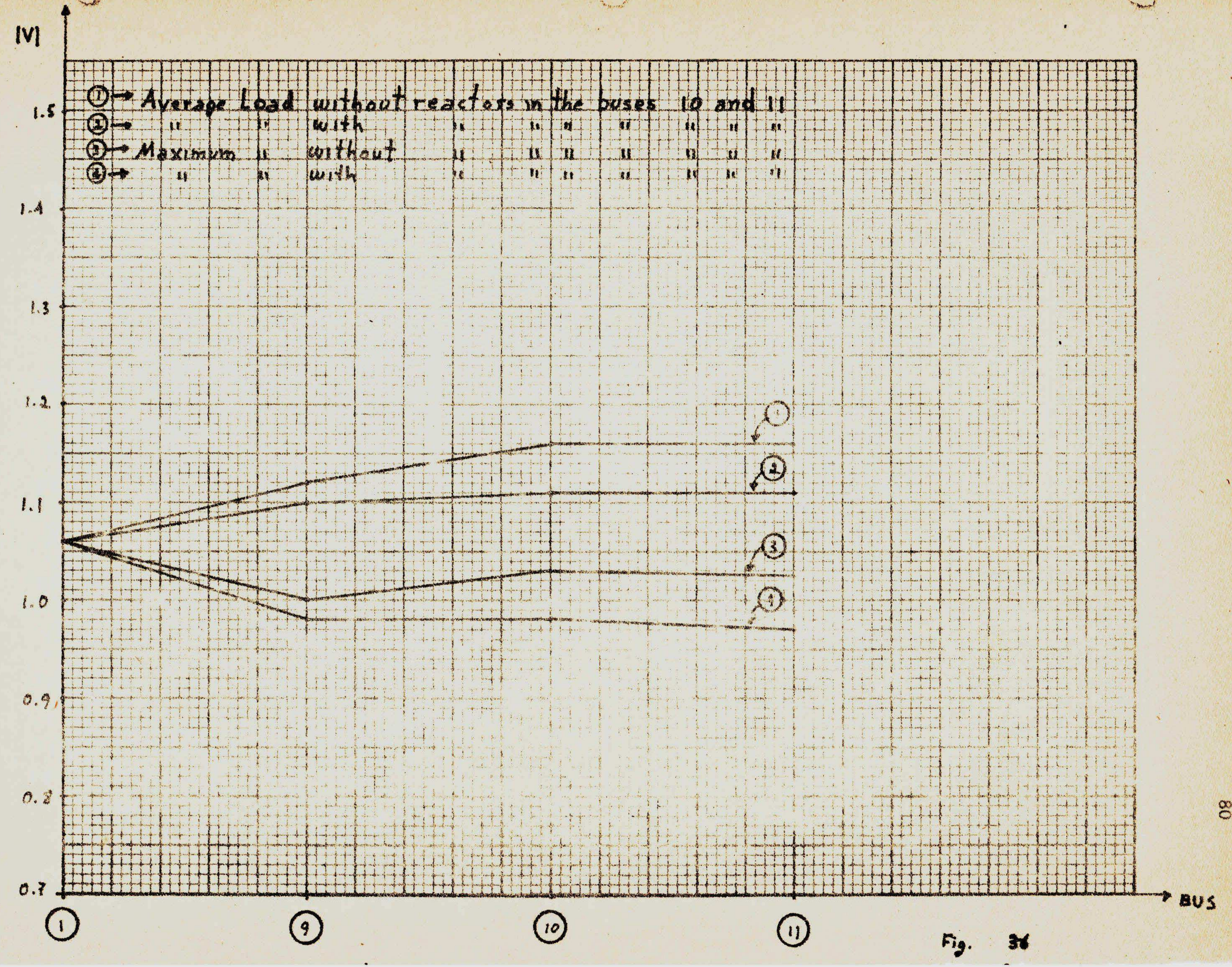

|V|
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
① → Average Load without reactors in the buses 10 and 11
② → " " with " " " " " " "
③ → Maximum " without " " " " " " "
④ → " " with " " " " " " "
①
②
③
④
BUS
①
⑨
⑩
⑪

Fig. 36

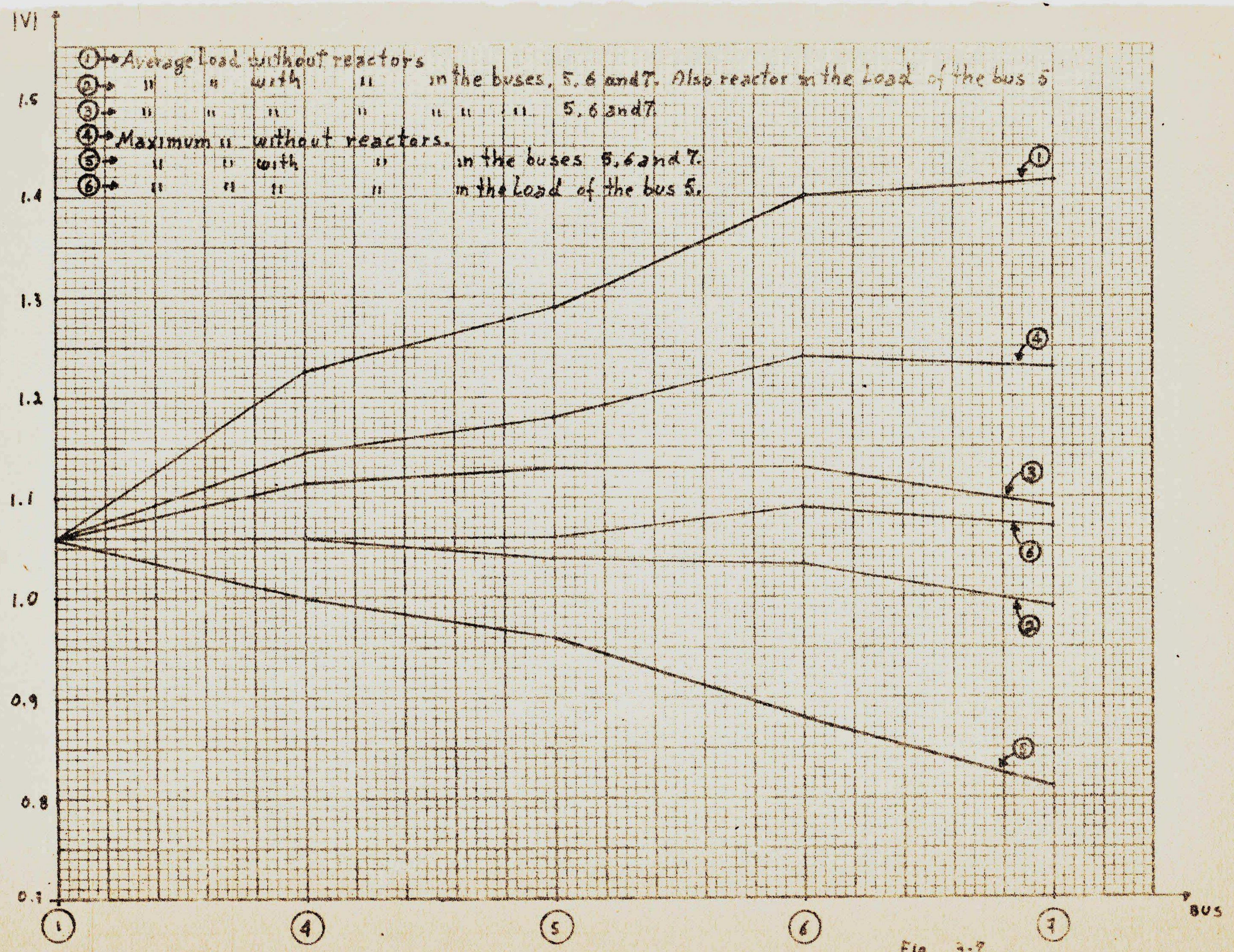
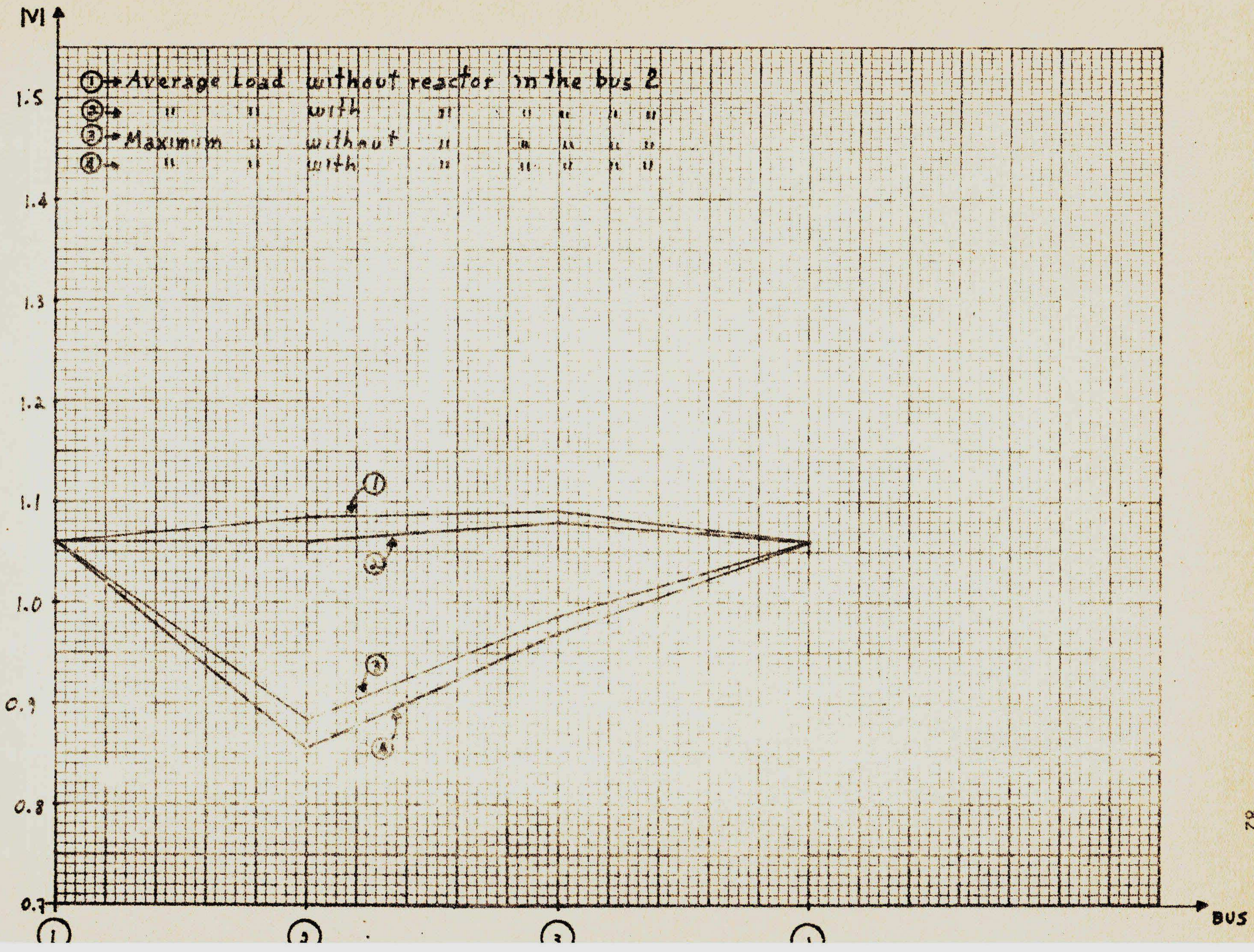

|V|
① → Average Load without reactors
② → " " with " in the buses, 5, 6 and 7. Also reactor in the Load of the bus 5
③ → " " " " " " " 5, 6 and 7.
④ → Maximum " without reactors.
⑤ → " " with " in the buses 5, 6 and 7.
⑥ → " " " " in the Load of the bus 5.
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
①
④
⑤
⑥
⑦
BUS
①
②
③
④
⑤
⑥

Fig. 3·7


M
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
BUS
①
②
③
①
① → Average Load without reactor in the bus 2
② → " " with " " " " "
③ → Maximum " without " " " " "
④ → " " with " " " " "
①
②
③
④

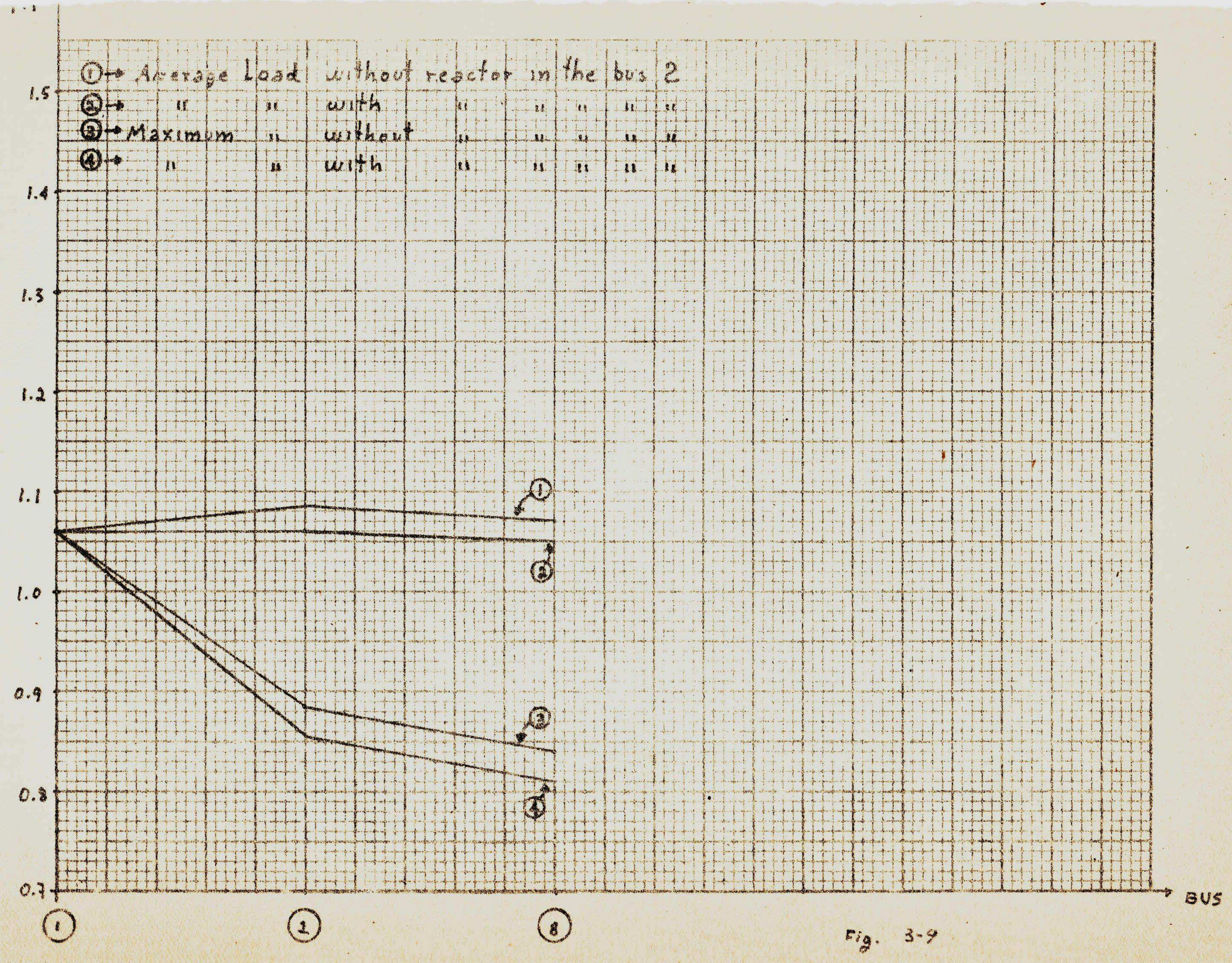

①→ Average Load without reactor in the bus 2
②→ " " with " " " " "
③→ Maximum " without " " " " "
④→ " " with " " " " "
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
BUS
①
②
⑧
①
②
③
④

Fig. 3-9

From all situations shown before, we can see easily that the best two limit conditions for the system are:

Lines set from PAULO AFONSO to FORTALEZA

I - Maximum load with reactor on, only in the load of MILAGRES, Bus 5, Case e.

II - Average load with all reactors on, Case c.

Rest of the system

I - Maximum load with all reactors off, Case b.

II - Average load with all reactors on, Case c.

We can see better all these limit conditions separately in the following diagrams, Figures 3-10, 3-11, 3-12, 3-13 and 3-14.

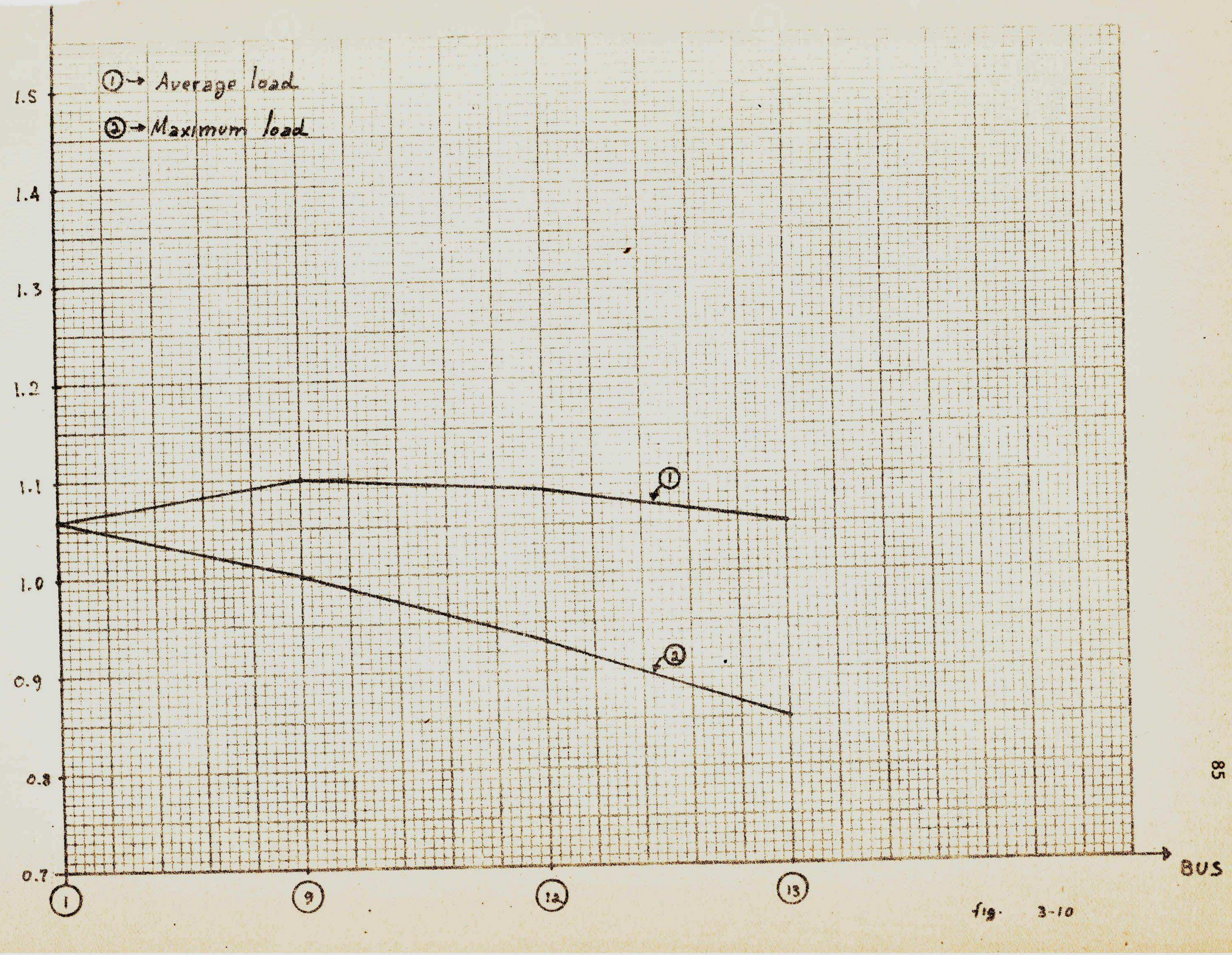
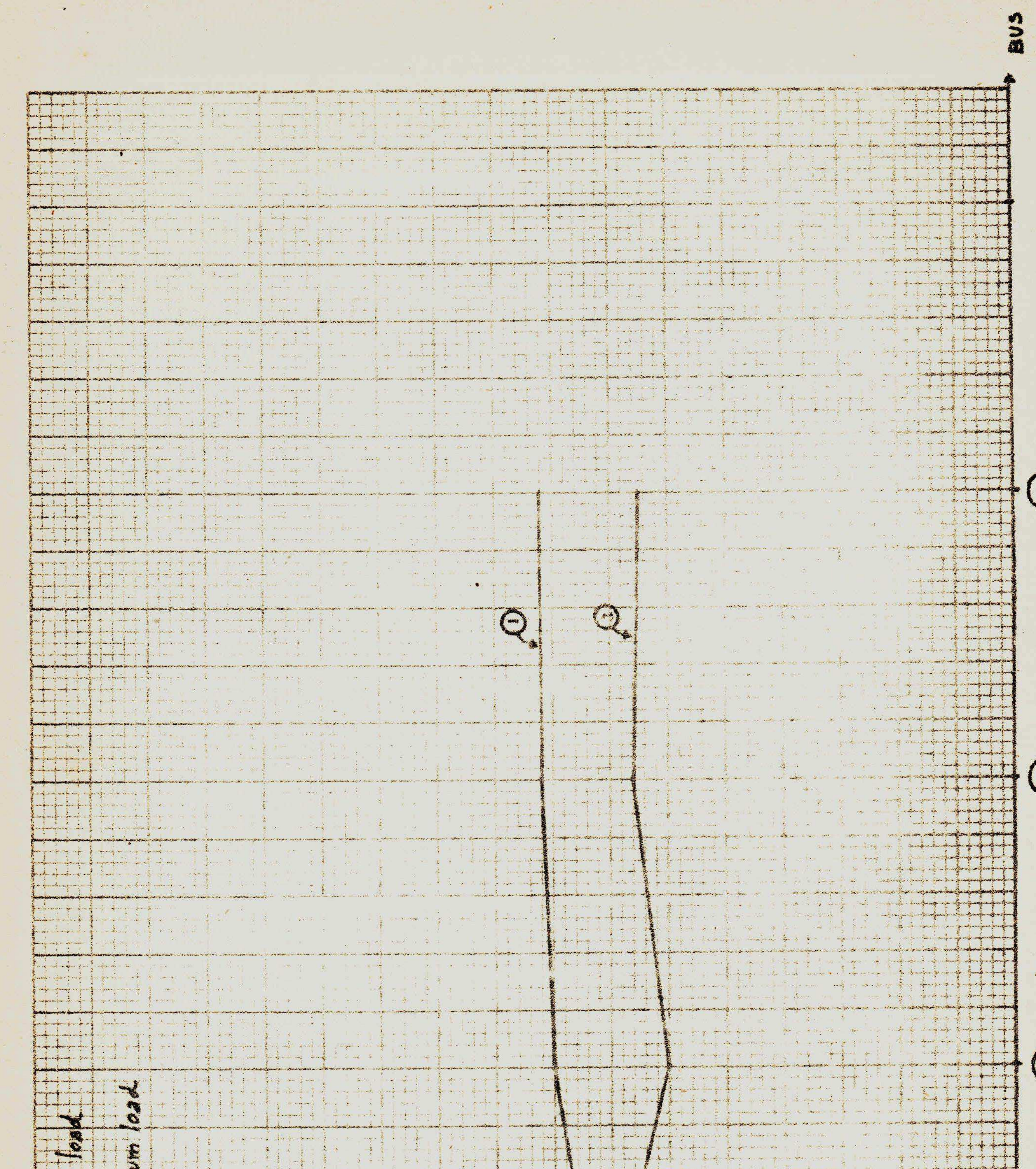

① → Average load
② → Maximum load
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
①
②
1
9
12
13
BUS

fig. 3-10


|V|
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
BUS
①
⑨
⑩
⑪
① → Average load
② → Maximum load
①
②

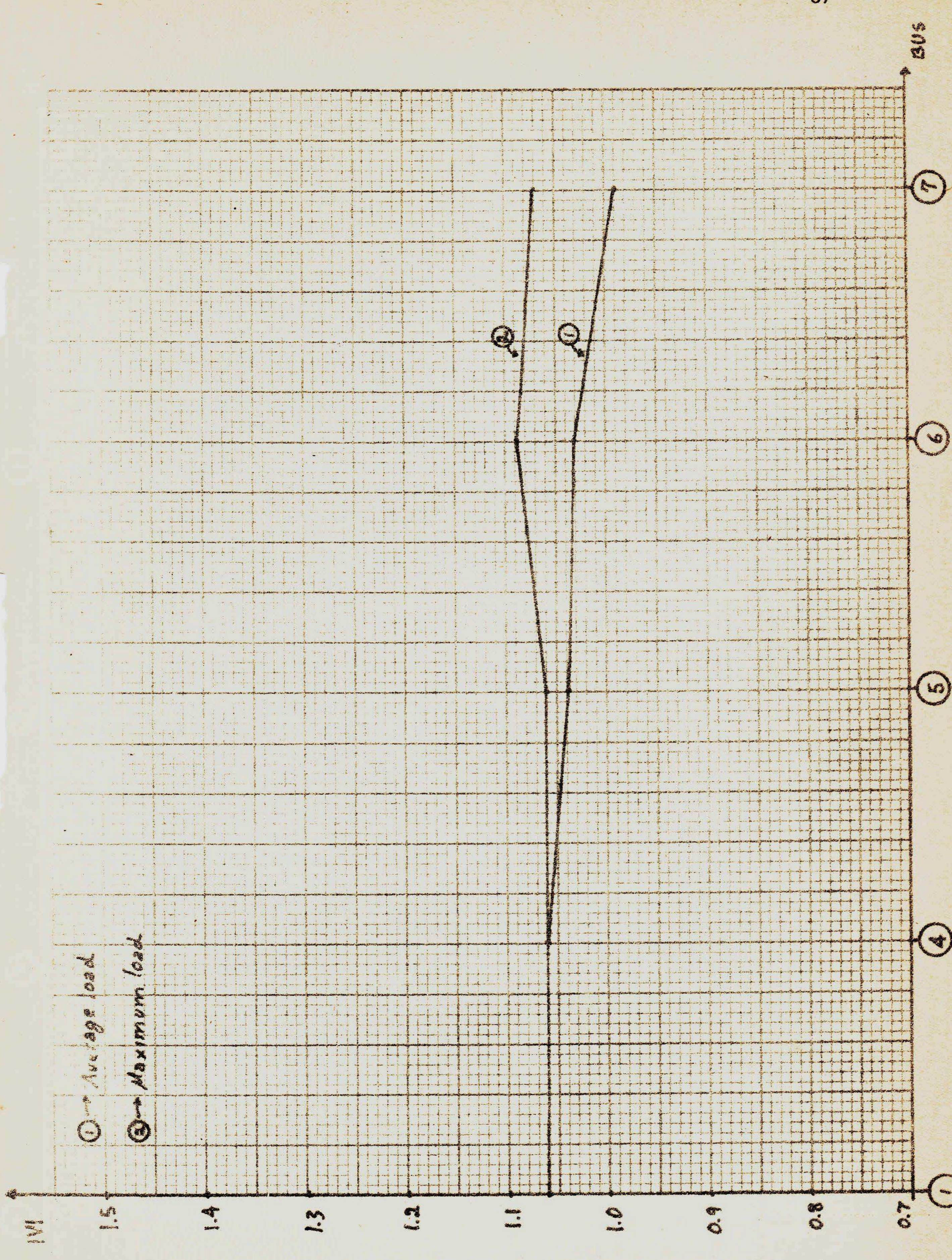
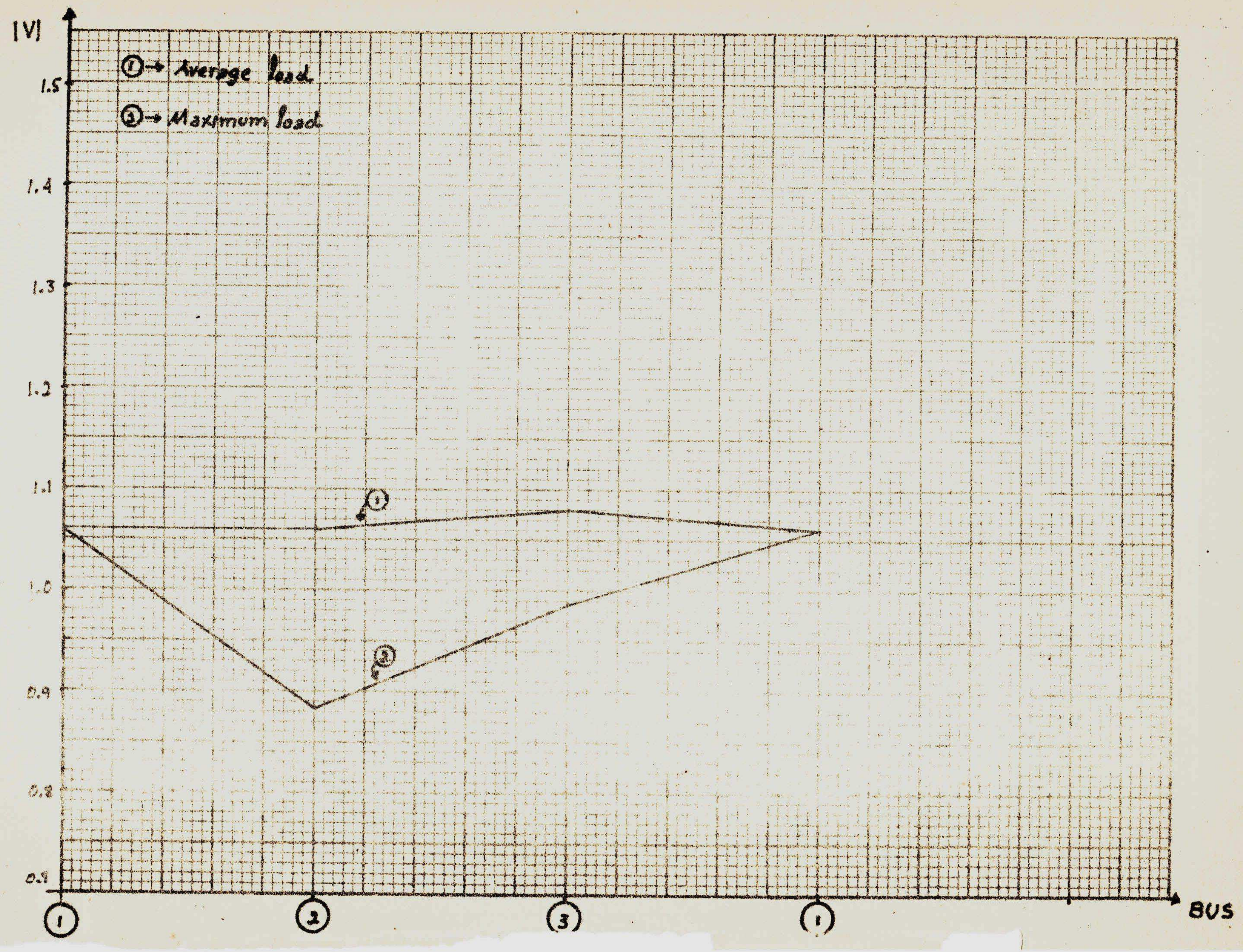

|V|
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
BUS
①
④
⑤
⑥
⑦
① → Average load
② → Maximum load
②
①

fig. 3-12


|V|
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
① → Average load
② → Maximum load
①
②
BUS
①
②
③
①

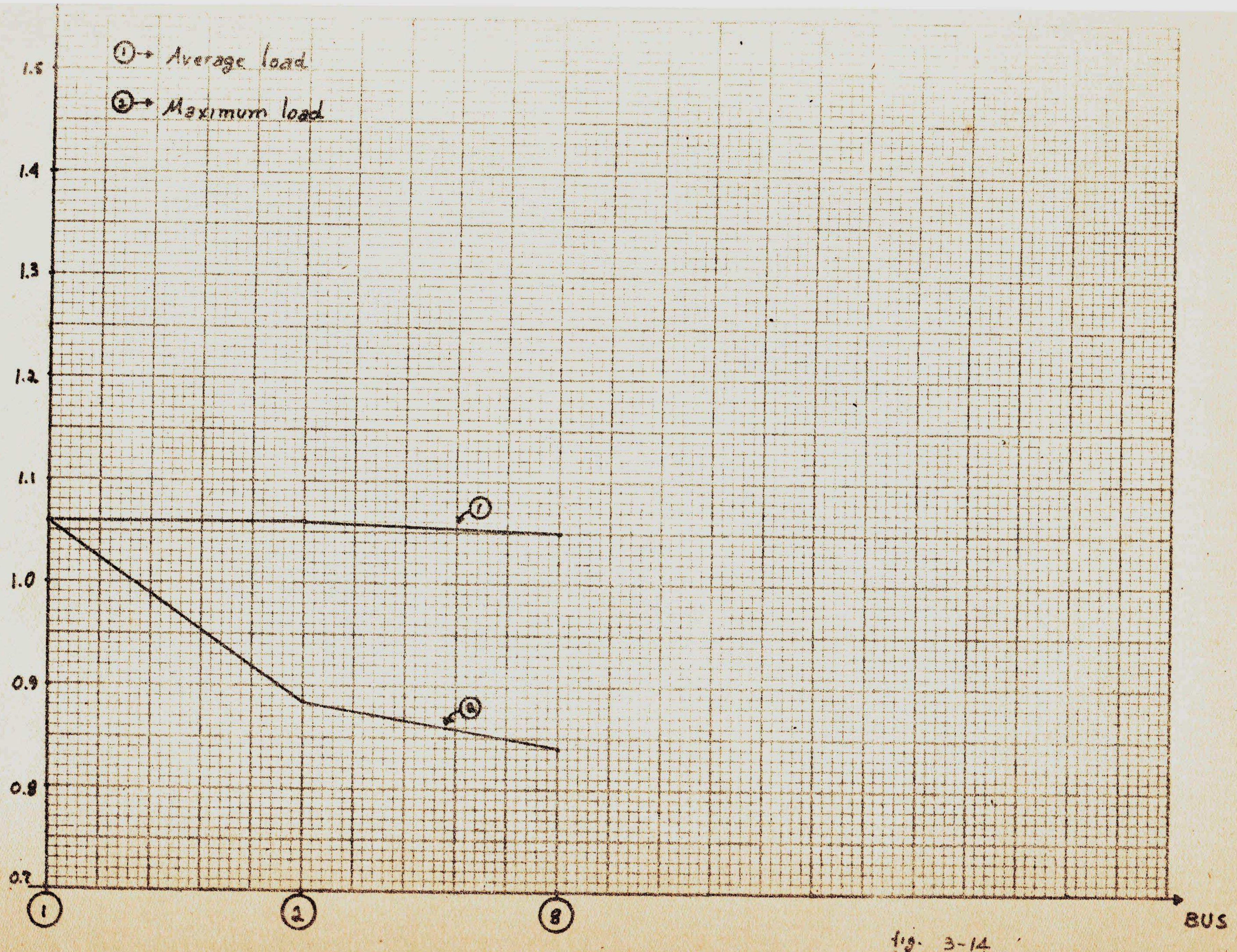

① → Average load
② → Maximum load
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
①
②
①
②
⑧
BUS

fig. 3-14

3.5.2 SYSTEM REGULATION STUDY

Now, from those best limits conditions of the system, we can talk about the system regulation study.

Regulation - by definition, the regulation at each bus of the system is:

$$\text{Regulation in percent} \rightarrow \text{Reg. } \% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100 \qquad (3\text{-}2\text{-}1)$$

where:

V_{NL} = Voltage magnitude without load (no load)

V_{FL} = Voltage magnitude with maximum load (full load).

But, in our case, we will take for V_{NL}, the voltage magnitude at minimum load, because it (no load) is not an actual condition in one power system.

On the other hand, we don't have the magnitude of the voltage at minimum load; we have the magnitude of the voltage at maximum load and average load.

Assume now:

V_{AL} = Voltage magnitude at average load

and V_{NL} = Voltage magnitude at minimum load if we assume that:

$$V_{AL} - V_{FL} \cong V_{NL} - V_{AL} \qquad (3\text{-}2\text{-}2)$$

In other words, we are assuming that the variation between the voltages at average load and maximum load is closely the same for minimum and average load.

Then, we have that (from the equation (3-2-2)):

$$V_{NL} \cong 2V_{AL} - V_{FL} \qquad (3\text{-}2\text{-}3)$$

Now, substituting (3-2-3) into (3-2-1) we have:

$$\text{Reg. } \% = \frac{2\ (V_{AL} - V_{FL})}{V_{FL}} \times 100 \qquad (3\text{-}2\text{-}4)$$

where all the values of (3-2-4) are known.

By use of one small program, which follows, we can determine the regulation of the system for the two best limit conditions.

PROGRAM

```
$JOB        WATFIVE
            DO 10 I = 2, 13
            READ, VAL, VFL
            REG = 200.0 *(VAL - VFL)/VFL
            PRINT, 'BUS NO.', I,' REGULATION', REG,' PERCENT'
         10 CONTINUE
            STOP
            END
$ENTRY
```

From the output of the above program we can get:

BUS	REGULATION IN PERCENT
2 - CATU	40.43
3 - ITABAIANA	19.21
4 - BOM NOME	-0.99

5 - MILAGRES	-3.23
6 - BANABUIU	-10.30
7 - FORTALEZA	-14.08
8 - MATATU	49.80
9 - ANGELIN	21.84
10 - CAMPINA GRANDE	15.51
11 - SANTA CRUZ	14.43
12 - RECIFE	32.30
13 - GOIANINHA	45.60

So, from the values of the regulation found above, we can see that most of the buses have greater than 10% regulation in magnitude, and we will try and make them within that limit, i.e. less than 10%.

System regulation improving

Verifying the reactive flow in the lines and the voltages of the buses, we can try to control them by reactive injection in some buses and obtain better regulation for the system.

System with average load

BUS 2 - CATU - 0.1 of inductive reactives.

OBSERVATION

The values of reactive power have the same magnitude as admittance. Hence, the values may be given in power or admittance indifferently.

BUS 4 - BOM NOME - 0.1 of inductive reactive power.

BUS 5 - MILAGRES - 0.1 directly on the line and 0.[illegible] in the load of inductive reactive power.

BUS 6 - BANABUIU - 0.1 of inductive reactive power.

BUS 7 - FORTALEZA - 0.15 of inductive reactive power.

BUS 9 - ANGELIN - Transformer in nominal tap from ANGELIN to CAMPINA GRANDE, BUS 10.

BUS 10 - CAMPINA GRANDE- 0.25 of inductive reactive power.

BUS 11 - SANTA CRUZ - 0.05 of inductive reactive power.

BUS 12 - RECIFE - 0.05 of inductive reactive power. Transformer in the tap 0.975 from RECIFE to GOIANINHA.

System with maximum load

BUS 5 - MILAGRES - 0.30 of inductive reactive power.

BUS 8 - MATATU - 0.45 of capacitive reactives.

BUS 9 - ANGELIN - Transformer in nominal tap from ANGELIN to CAMPINA GRANDE, BUS 10.

BUS 10 - CAMPINA GRANDE- 0.10 of inductive reactive power.

Bus 12 - RECIFE - 0.40 of capacitive reactive power. Transformer in the tap 0.95 from RECIFE to GOIANINHA, BUS 13.

BUS 13 - GOIANINHA - 0.10 of capacitive reactive power.

OBSERVATIONS

In the injection of reactive power on the buses of RECIFE and MATATU, synchronous capacitors must be used because the big variation of reactive powers at the limit conditions.

Results of the voltages and line flows at the shown conditions

1) System with average load.

2) System with maximum load.

```
BUS NAME  |          |          | BUS VOLTAGE|           |     BUS POWER      |
          |   REAL   |IMAGINARY | MAGNITUDE  |   ANGLE   |    REAL  | REACTIVE|

P.AFONSO  | 1.06000  |  0.00000 |  1.06000   |  0.00000  |  3.30706 | -2.32412|
          | 1.05082  | -0.15406 |  1.06205   | -8.34059  | -0.46810 | -0.22800|
ITABAIA.  | 1.07602  | -0.08644 |  1.07949   | -4.59271  | -0.16150 | -0.07870|
B.NOME    | 1.07162  | -0.06367 |  1.07352   | -3.40987  | -0.07410 | -0.01170|
MILAGRES  | 1.05576  | -0.08899 |  1.05951   | -4.81925  | -0.07060 | -0.33430|
BANABUIU  | 1.05534  | -0.15137 |  1.06710   | -8.17692  | -0.00920 | -0.00400|
FORTALE.  | 1.01905  | -0.18559 |  1.03581   |-10.32148  | -0.27300 | -0.13290|
MATATU    | 1.03316  | -0.17727 |  1.04826   | -9.73608  | -0.36860 | -0.17950|
ANGELIN   | 1.07906  | -0.12134 |  1.08592   | -6.44236  | -0.22950 | -0.11130|
C.GRANDE  | 1.05118  | -0.15118 |  1.06199   | -8.18410  | -0.08230 | -0.04010|
S.CRUZ    | 1.03715  | -0.16637 |  1.05041   | -9.11302  | -0.13430 | -0.06540|
RECIFE    | 1.04687  | -0.18473 |  1.06304   |-10.00750  | -1.11430 | -0.55550|
GOIANIN.  | 1.03248  | -0.21837 |  1.05532   |-11.94225  | -0.26590 | -0.12950|
```

REAL REACTIVE

TRANSMISSION NETWORK LOSS 0.1075 -4.19589

```
           *****LINE FLOWS *****

FROM BUS  TO BUS          REAL POWER  REACTIVE POWER

P.AFONSO  CATU              0.498497     -0.379002
P.AFONSO  ITABAIA.          0.528056     -0.352029
P.AFONSO  B.NOME            0.388103     -0.294049
P.AFONSO  ANGELIN           1.892324     -1.297463
CATU      P.AFONSO         -0.485014     -0.183131
CATU      ITABAIA.         -0.353526     -0.207546
CATU      MATATU            0.370304      0.049596
ITABAIA.  P.AFONSO         -0.519515      0.093729
ITABAIA.  CATU              0.357961     -0.172446
B.NOME    P.AFONSO         -0.383315     -0.003386
B.NOME    MILAGRES          0.359703     -0.002710
MILAGRES  B.NOME           -0.357810     -0.148776
MILAGRES  BANABUIU          0.297334     -0.297753
BANABUIU  MILAGRES         -0.293964     -0.110149
BANABUIU  FORTALE.          0.275943     -0.007718
FORTALE.  BANABUIU         -0.273133     -0.293979
MATATU    CATU             -0.368782     -0.179370
ANGELIN   P.AFONSO         -1.849278      0.242065
ANGELIN   C.GRANDE          0.219088     -0.079838
ANGELIN   RECIFE            1.400753     -0.274136
C.GRANDE  ANGELIN          -0.217327     -0.275768
C.GRANDE  S.CRUZ            0.135210     -0.066404
S.CRUZ    C.GRANDE         -0.134349     -0.120565
RECIFE    ANGELIN          -1.395208     -0.795828
RECIFE    GOIANIN.          0.207033     -0.055130
GOIANIN.  RECIFE           -0.204399      0.012470
```

[illegible] System with maximum load.

```
BUS NAME  |                        BUS VOLTAGE                       |        BUS POWER
          |     REAL    IMAGINARY   MAGNITUDE      ANGLE             |
          |
[illegible] 1.06000     0.00000     1.06000       0.00000           7.13306   -1.49996
[illegible] 0.97788    -0.29822     1.02234     -16.96011          -0.88550   -0.43130
ITABAIA., 1.03499    -0.16681     1.04834      -9.15583          -0.36150   -0.08790
[illegible] 1.05105    -0.16282     1.06359      -8.80571          -0.05780   -0.02820
MILAGRES  1.03229    -0.23382     1.05844     -12.76262          -0.16500   -0.38030
BANABUIU  1.01392    -0.40087     1.09029     -21.57228          -0.02350    0.00660
FORTALE.  0.94892    -0.49289     1.06929     -27.44839          -0.70330   -0.12650
MATATU    0.96072    -0.35402     1.02387     -20.22827          -0.73000   -0.35550
ANGELIN   1.02350    -0.24402     1.05219     -13.41007          -0.53000   -0.15610
C.GRANDE  1.01502    -0.32618     1.06614     -17.81511          -0.15860   -0.02320
S.CRUZ    0.99735    -0.36535     1.06235     -20.14400          -0.28650   -0.01185
RECIFE    0.95522    -0.37351     1.02565     -21.35643          -2.22350   -0.69066
GOIANIN.  0.92851    -0.45447     1.03377     -26.07990          -0.54600   -0.16790
```

	REAL	ACTIVE
TRANSMISSION NETWORK LOSS	0.46186	-3.95277

```
                 *****LINE FLOWS*****

FROM BUS   TO BUS            REAL POWER  REACTIVE POWER

P.AFONSO  CATU                 1.003590      -0.233003
P.AFONSO  ITABAIA.             1.084604      -0.185626
P.AFONSO  B.NOME               1.030528      -0.305919
P.AFONSO  ANGELIN              4.013364      -0.775101
CATU      P.AFONSO            -0.949431      -0.069745
CATU      ITABAIA.            -0.676171      -0.135538
CATU      MATATU               0.738101      -0.206008
ITABAIA.  P.AFONSO            -1.053158       0.060825
ITABAIA.  CATU                 0.691655      -0.148741
B.NOME    P.AFONSO            -0.999692       0.138300
B.NOME    MILAGRES             0.942184      -0.165728
MILAGRES  B.NOME              -0.929525       0.069956
MILAGRES  BANABUIU             0.764593      -0.450098
BANABUIU  MILAGRES            -0.740526       0.140919
BANABUIU  FORTALE.             0.717328      -0.133902
FORTALE.  BANABUIU            -0.703441      -0.126686
MATATU    CATU                -0.730000       0.116240
ANGELIN   P.AFONSO            -3.841563       0.449299
ANGELIN   C.GRANDE             0.456246      -0.329365
ANGELIN   RECIFE               2.855295      -0.276081
C.GRANDE  ANGELIN             -0.448911       0.011942
C.GRANDE  S.CRUZ               0.290303      -0.148828
S.CRUZ    C.GRANDE            -0.286502      -0.011850
RECIFE    ANGELIN             -2.788560      -0.332730
RECIFE    GOIANIN.             0.565055       0.062864
GOIANIN.  RECIFE              -0.545996      -0.061031
```

Now we will plot the voltages versus Bus Number for the new two limit conditions (average and maximum load), Figures 3-15, 3-16, 3-17, 3-18 and 3-19.

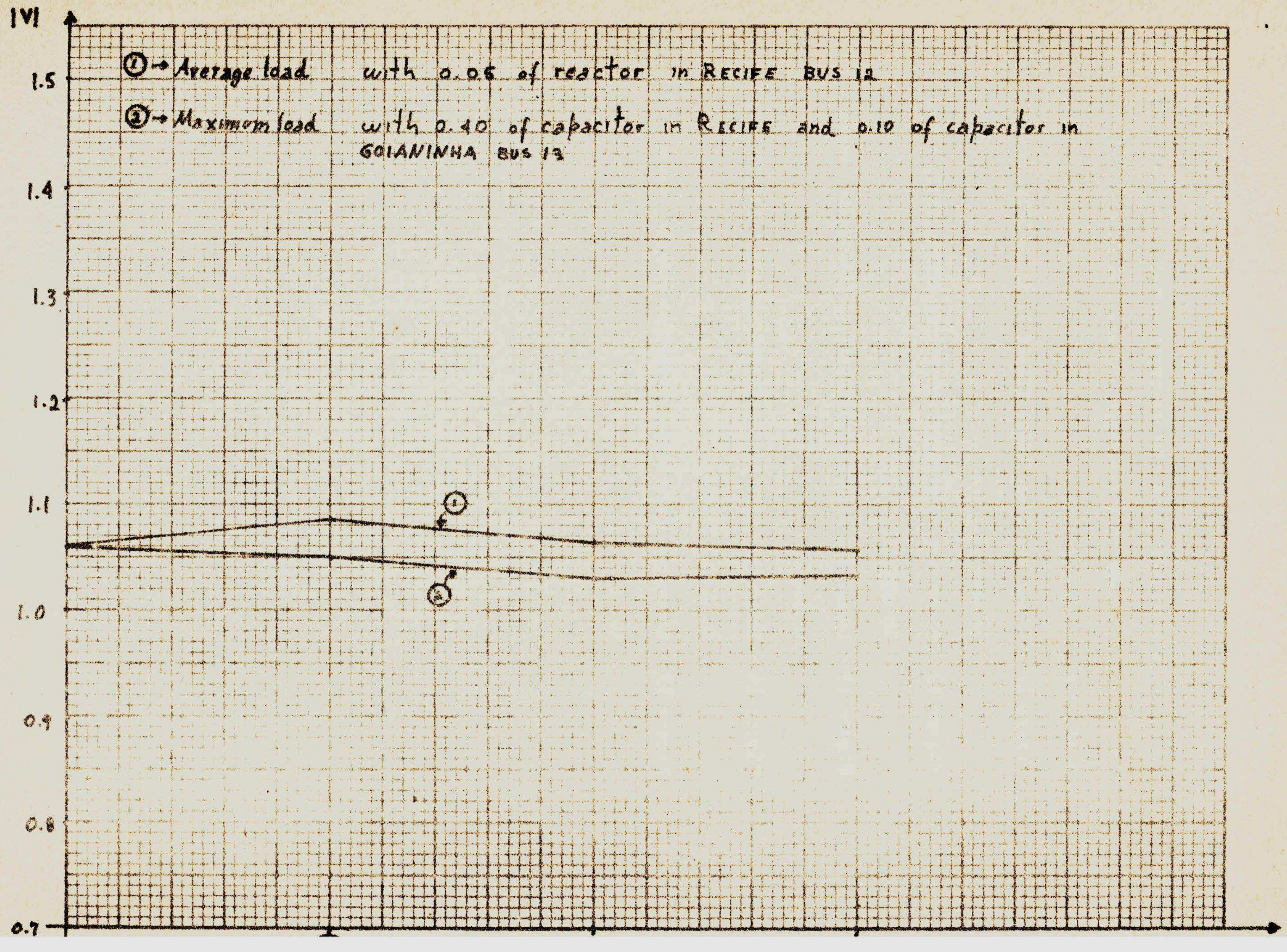
Figure 11 - Lines set from PAULO AFONSO to BOIANINHA, buses 1, 9, 12 and 13.

Figure 12 - Lines set from PAULO AFONSO to SANTA CRUZ, buses 1, 9, 10 and 11.

Figure 13 - Lines set from PAULO AFONSO to FORTALEZA, buses 1, 4, 5, 6 and 7.

Figure 14 - Lines set from PAULO AFONSO to PAULO AFONSO loop by 1, 2 and 3 buses.

Figure 15 - Lines set from PAULO AFONSO to MATATU, buses 1, 2 and 8.


|V|
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
① → Average load with 0.05 of reactor in RECIFE BUS 12
② → Maximom load with 0.40 of capacitor in RECIFE and 0.10 of capacitor in GOIANINHA BUS 13
①
②

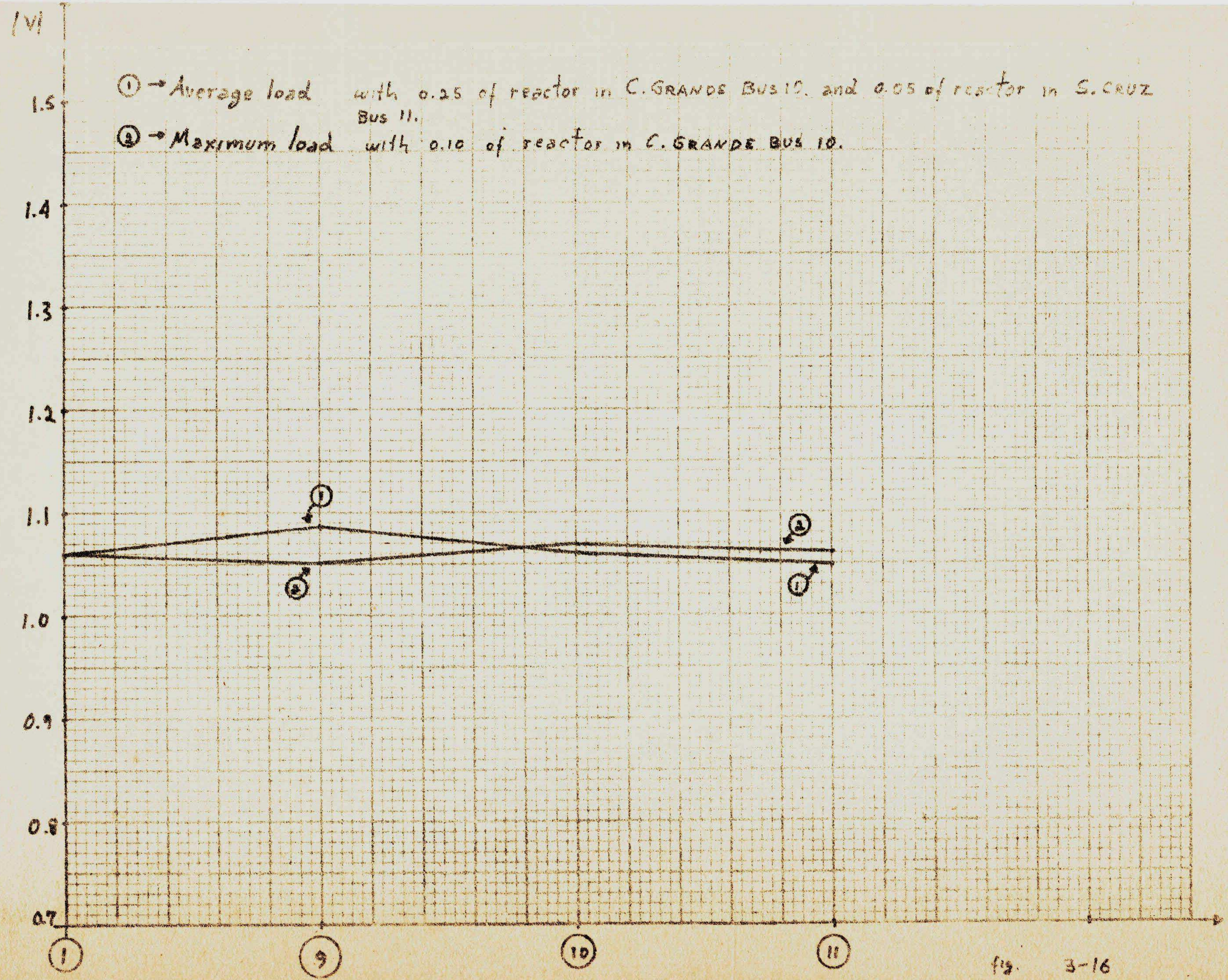
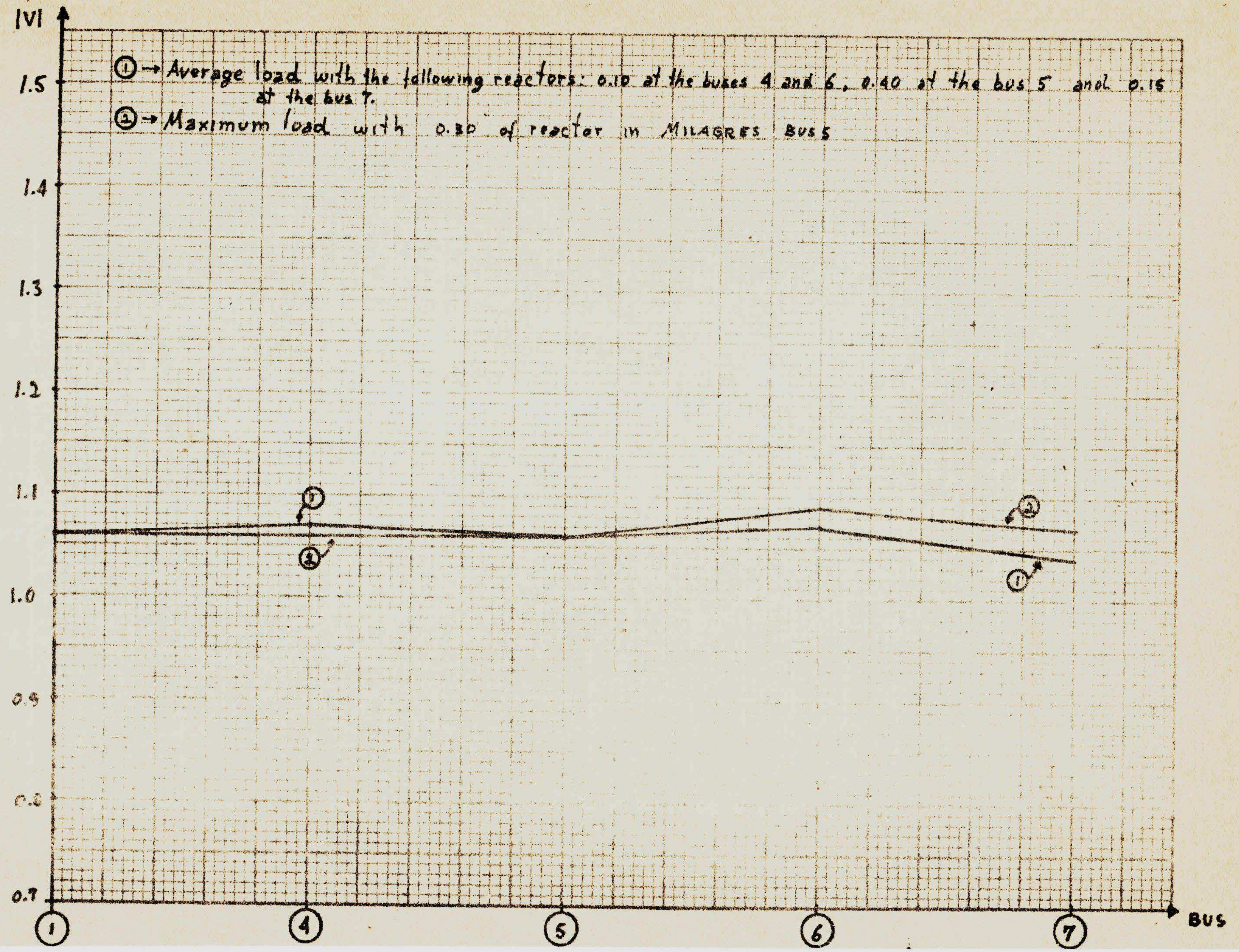

|V|
① → Average load with 0.25 of reactor in C. GRANDE Bus 10. and 0.05 of reactor in S. CRUZ Bus 11.
② → Maximum load with 0.10 of reactor in C. GRANDE Bus 10.
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
①
②
②
①
①
⑨
⑩
⑪

fig. 3-16


|V|
①→ Average load with the following reactors: 0.10 at the buses 4 and 6, 0.40 at the bus 5 and 0.15 at the bus 7.
②→ Maximum load with 0.30 of reactor in MILAGRES BUS 5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
①
②
①
④
⑤
⑥
⑦
BUS

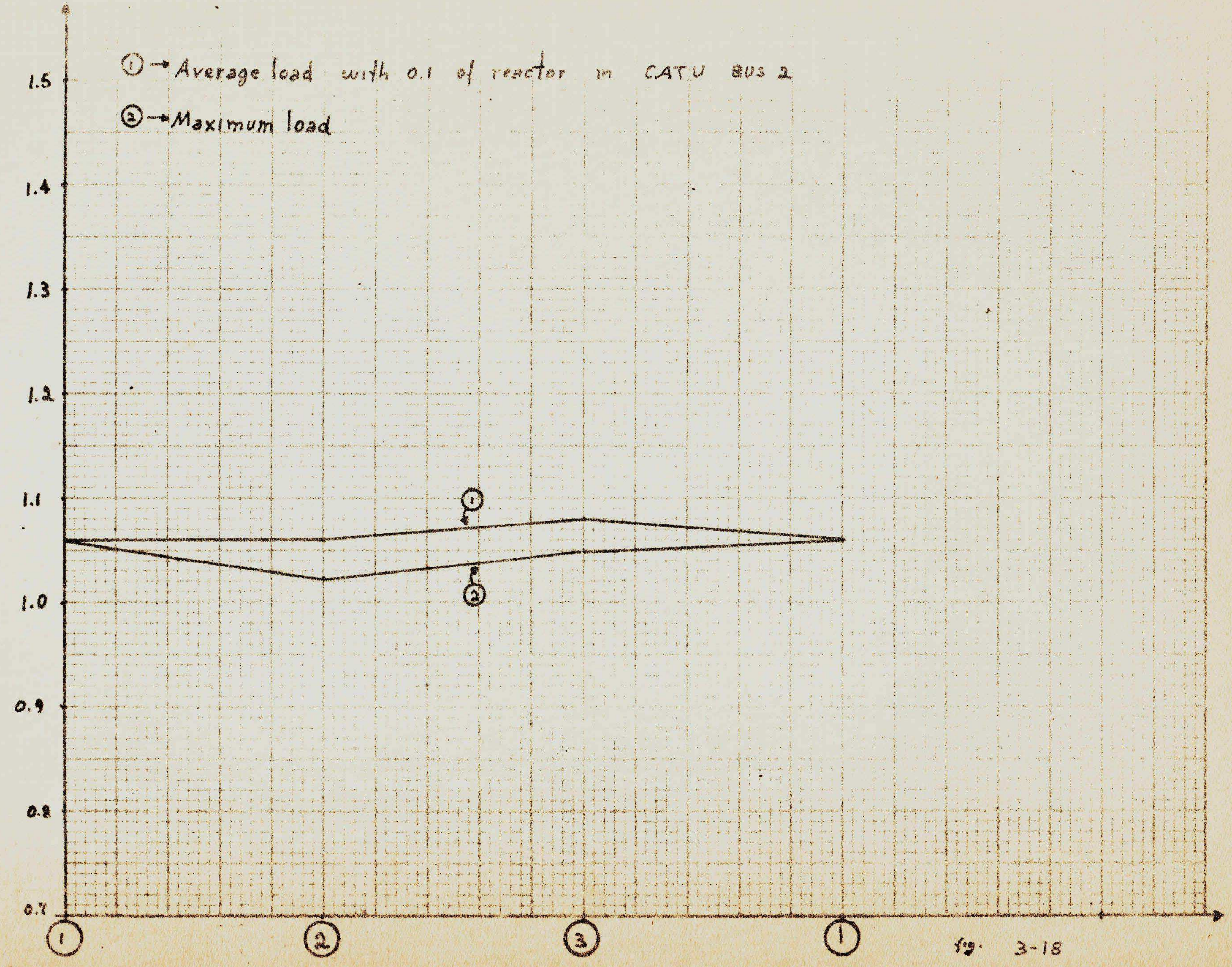
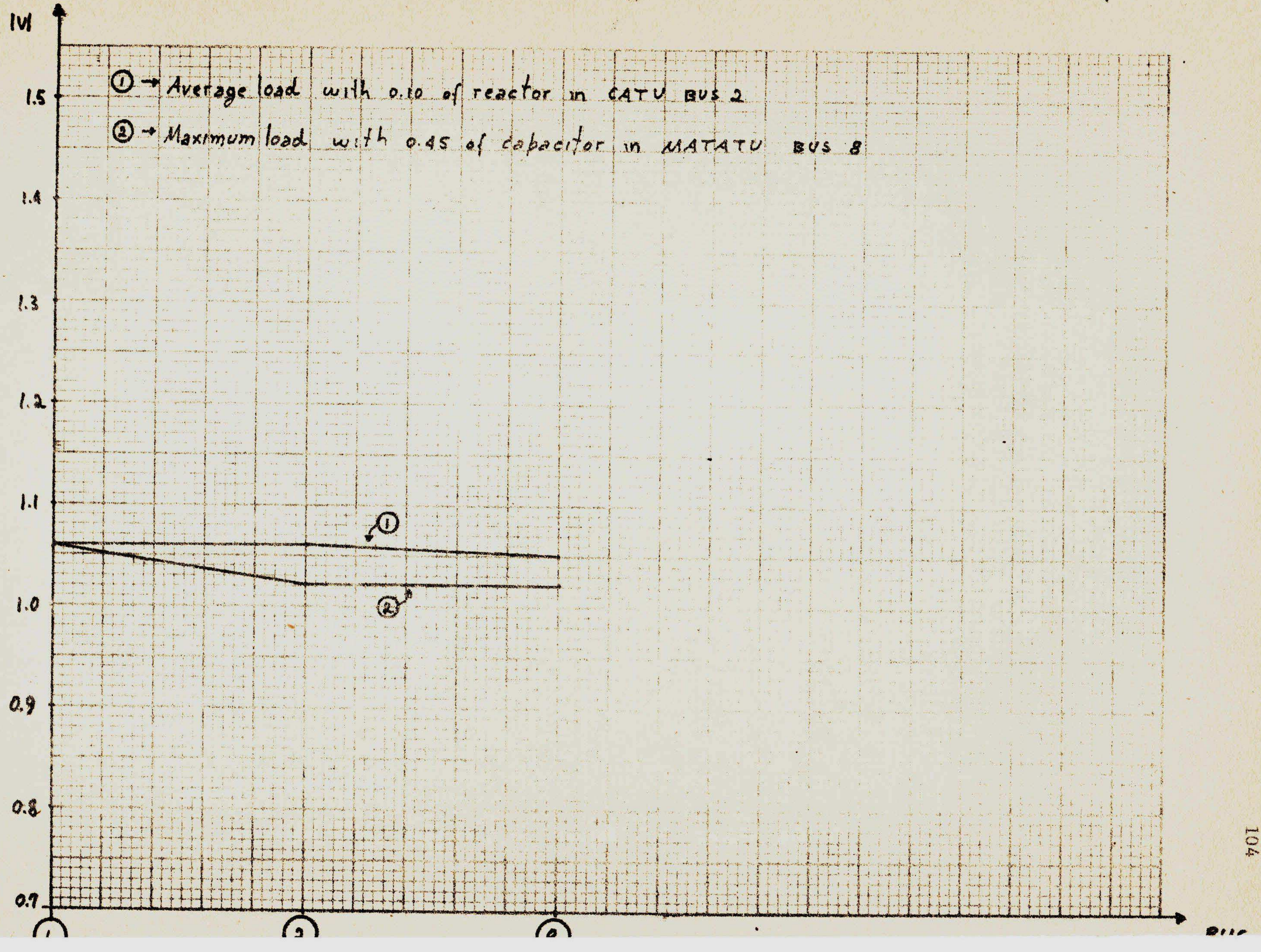
 ① → Average load with 0.1 of reactor in CATU BUS 2
② → Maximum load
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
①
②
③
①
①
②

fig. 3-18


|V|
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
① → Average load with 0.10 of reactor in CATU BUS 2
② → Maximum load with 0.45 of capacitor in MATATU BUS 8
①
②

New regulation of the system

From the two new limit conditions after the shown changes, we can calculate new regulation of the system which is:

BUS	REGULATION IN PERCENT
2 - CATU	7.77
3 - ITABAIANA	5.95
4 - BOM NOME	1.32
5 - MILAGRES	0.21
6 - BANABUIU	-4.36
7 - FORTALEZA	-6.25
8 - MATATU	4.77
9 - ANGELIN	6.40
10 - CAMPINA GRANDE	-0.77
11 - SANTA CRUZ	-2.26
12 - RECIFE	7.29
13 - GOIANINHA	4.16

Now we have all the bus regulations below 10% which is one good condition for the transmission system.

PART TWO

4. SINGLE LINE TO GROUND FAULT IN A SYSTEM WITH ISOLATED NEUTRAL, USING GRAPH THEORY

In this section we shall study the system in which some generators with isolated neutral exist. The method is shown using an example. The example system has tree buses and two generators. Symmetrical components are used for solving the system.

In Figure 4-1 we have the single line diagram where the generator "A" has isolated neutral, and the generator "B" is solid grounded.

Transmission system data

Element	Bus Code	Self Impedance		
	From Bus to Bus	Positive	Negative	Zero
1	1 - 2	j0.06	j0.06	j0.10
2	1 - 2	j0.06	j0.06	j0.12
3	2 - 3	j0.08	j0.08	j0.14
4	1 - 3	j0.13	j0.13	j0.17

Generator system data

Generator	Self Impedance			Voltage Source		
	Positive	Negative	Zero	Positive	Negative	Zero
A	j0.25	j0.15	j0.03	$\sqrt{3} \times 1.05 + j0$	0	0
B	j0.20	j0.12	j0.02	$\sqrt{3} \times 1.05 + j0$	0	0

For a fault at bus 2 we will determine the short circuit voltages and currents of the system using Z_{BUS} method.

The transmission system graph and generator system graph are shown in Figures 4-2, where:

Figure 4-2a - system graph (transmission system)

4-2b - graph of the generator B

4-2c - graph of the generator A at positive and negative sequence

4-2d - graph of the generator A at zero sequence.

We will derive terminal equations for the system at the form

$$V_{BUS}^{0,1,2} = Z_{BUS}^{0,1,2} + V_{0}^{0,1,2} \qquad (4-1)$$

Initially we will derive a terminal equation for the transmission system with bus 2 as reference.

The Figures 4-3a, b and c show the terminal graph of the system, the augmented source drivers and the augmented graph of the transmissions system respectively.

From the augmented transmission system graph we will write the fundamental circuit equations choosing the elements 1 and 3 as tree. Then, we have:

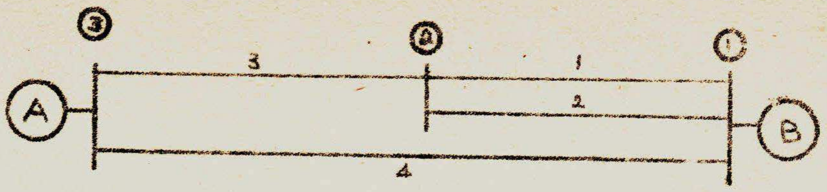
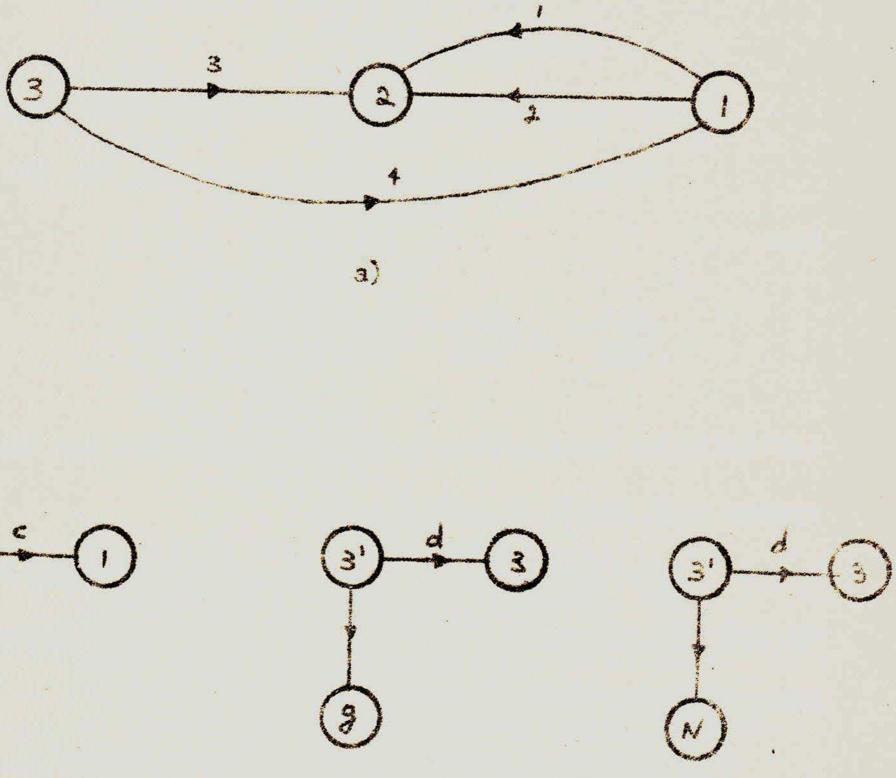


Figure 4-1

Single line diagram of the system.



a)

b)

c)

d)

Figure 4-2

a) Graph of the transmission system.

b) Graph of the generator B.

c) Graph of the generator A at positive and negative sequence.

d) Graph of the generator A at zero sequence.

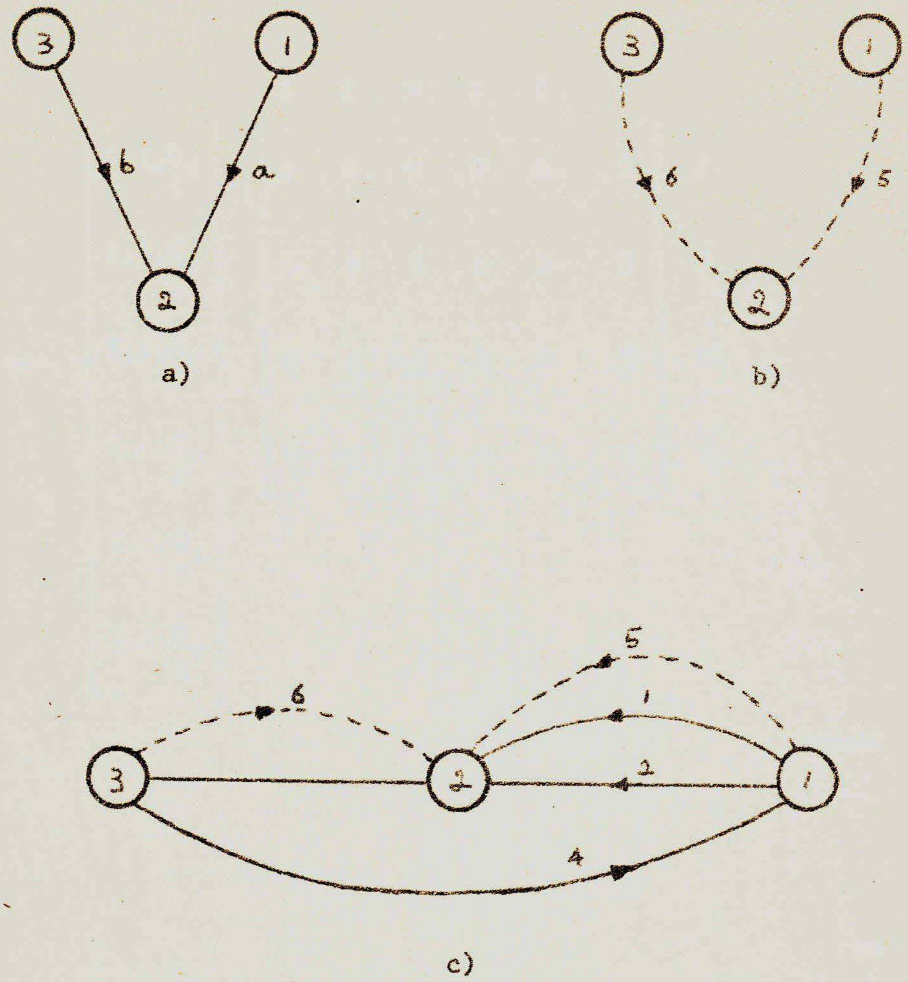


Figure 4-3

a) Terminal graph of the transmission system.

b) Augmented source drivers.

c) Augmented system graph.

$$\begin{array}{c} \\ 2 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 3 & 2 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -U & 0 & U & 0 & 0 & 0 \\ U & -U & 0 & U & 0 & 0 \\ U & 0 & 0 & 0 & U & 0 \\ 0 & -U & 0 & 0 & 0 & U \end{bmatrix} \end{array} \begin{bmatrix} V_1^{0,1,2} \\ V_3^{0,1,2} \\ V_2^{0,1,2} \\ V_4^{0,1,2} \\ V_5^{0,1,2} \\ V_6^{0,1,2} \end{bmatrix} = 0 \qquad (4\text{-}2)$$

where

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ U & 0 \\ 0 & U \end{bmatrix} \begin{bmatrix} V_5^{0,1,2} \\ V_6^{0,1,2} \end{bmatrix} = - \begin{bmatrix} -U & 0 & U & 0 \\ U & -U & 0 & U \\ -U & 0 & 0 & 0 \\ 0 & -U & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^{0,1,2} \\ V_3^{0,1,2} \\ V_2^{0,1,2} \\ V_4^{0,1,2} \end{bmatrix} \qquad (4$$

From the system elements we have the terminal equations:

$$\begin{bmatrix} V_1^{0,1,2} \\ V_3^{0,1,2} \\ V_2^{0,1,2} \\ V_4^{0,1,2} \end{bmatrix} = \begin{bmatrix} Z_{11}^{0,1,2} & 0 & 0 & 0 \\ 0 & Z_{33}^{0,1,2} & 0 & 0 \\ 0 & 0 & Z_{22}^{0,1,2} & 0 \\ 0 & 0 & 0 & Z_{44}^{0,1,2} \end{bmatrix} \begin{bmatrix} I_1^{0,1,2} \\ I_3^{0,1,2} \\ I_2^{0,1,2} \\ I_4^{0,1,2} \end{bmatrix} \qquad (4\text{-}4)$$

Substituting equation (4-4) into equation (4-3) and expressing the brench currents in terms of chord currents we have:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ U & 0 \\ 0 & U \end{bmatrix} \begin{bmatrix} V_5^{0,1,2} \\ V_6^{0,1,2} \end{bmatrix} = - \begin{bmatrix} -U & 0 & U & 0 \\ U & -U & 0 & U \\ -U & 0 & 0 & 0 \\ 0 & -U & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_{11}^{0,1,2} & 0 & 0 & 0 \\ 0 & Z_{33}^{0,1,2} & 0 & 0 \\ 0 & 0 & Z_{22}^{0,1,2} & 0 \\ 0 & 0 & 0 & Z_{44}^{0,1,2} \end{bmatrix} \begin{bmatrix} -U & U & -U & 0 \\ 0 & -U & 0 & -U \\ U & 0 & 0 & 0 \\ 0 & U & 0 & 0 \end{bmatrix} \begin{bmatrix} I_2^{0,1,2} \\ I_4^{0,1,2} \\ I_5^{0,1,2} \\ I_6^{0,1,2} \end{bmatrix}$$

$$(4\text{-}5)$$

From the first two equations of equation (4-5) we can determine the values of $I_2^{0,1,2}$ and $I_4^{0,1,2}$. Substituting these in the rest of equation (4-5), we have a relation between

$$\begin{bmatrix} V_5^{0,1,2} \\ V_6^{0,1,2} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} I_5^{0,1,2} \\ I_6^{0,1,2} \end{bmatrix}$$

as:

$$\begin{bmatrix} V_5^{0,1,2} \\ V_6^{0,1,2} \end{bmatrix} = \begin{bmatrix} V_a^{0,1,2} \\ V_b^{0,1,2} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} I_5^{0,1,2} \\ I_6^{0,1,2} \end{bmatrix} = - \begin{bmatrix} I_a^{0,1,2} \\ I_b^{0,1,2} \end{bmatrix}$$

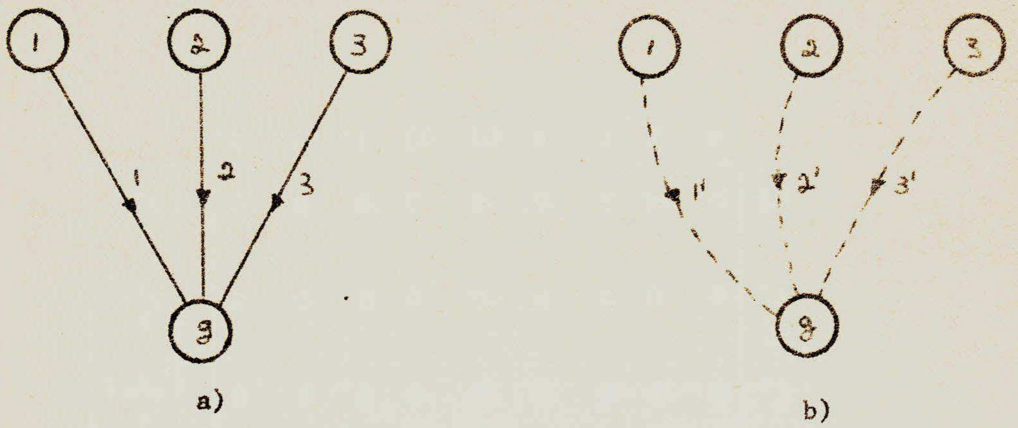
we have:

$$\begin{bmatrix} V_a^{0,1,2} \\ V_6^{0,1,2} \end{bmatrix} \begin{bmatrix} Z_{aa}^{0,1,2} & Z_{ab}^{0,1,2} \\ Z_{ab}^{0,1,2} & Z_{bb}^{0,1,2} \end{bmatrix} \begin{bmatrix} I_a^{0,1,2} \\ I_b^{0,1,2} \end{bmatrix} \qquad (4\text{-}6a)$$

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \\ V_b^0 \\ V_b^1 \\ V_b^2 \end{bmatrix} = j \left[\begin{array}{ccc:ccc} 0.0464 & 0 & 0 & 0.0209 & 0 & 0 \\ 0 & 0.0262 & 0 & 0 & 0.010 & 0 \\ 0 & 0 & 0.0262 & 0 & 0 & 0.010 \\ 0.0209 & 0 & 0 & 0.0862 & 0 & 0 \\ 0 & 0.010 & 0 & 0 & 0.0533 & 0 \\ 0 & 0 & 0.010 & 0 & 0 & 0.0533 \end{array}\right] \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \\ I_b^0 \\ I_b^1 \\ I_b^2 \end{bmatrix} \qquad (4\text{-}6b)$$

Now, we will derive for the generator and transmission system the terminal representation with the terminal graph of Figure 4-4a corresponding to positive and negative sequence.

For the overall graph of Figure 4-5, which is the union of the system terminal graph, Figure 4-3a and the augmented source drivers, Figure 4-4b and generator graph, Figures 4-2b, c; we have the fundamental circuit equations, choosing the elements a, b, c, d and 01 as tree.



a)

b)

Figure 4-4

a) Terminal graph.

b) Augmented source drivers.

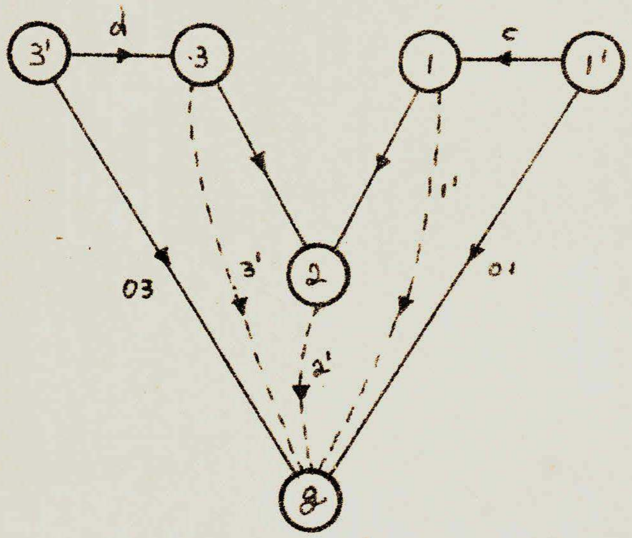


Figure 4-5

Overall graph.

$$\begin{array}{c|ccccccccc|}
 & a & b & c & c & 01 & 03 & 1' & 2' & 3' \\
03 & U & -U & U & -U & -U & U & 0 & 0 & 0 \\
1' & 0 & 0 & U & 0 & -U & 0 & U & 0 & 0 \\
2' & U & 0 & U & 0 & -U & 0 & 0 & U & 0 \\
3' & U & -U & U & 0 & -U & 0 & 0 & 0 & U
\end{array}
\begin{bmatrix} V_a^{1,2} \\ V_b^{1,2} \\ V_c^{1,2} \\ V_d^{1,2} \\ V_{01}^{1,2} \\ V_{03}^{1,2} \\ V_{1'}^{1,2} \\ V_{2'}^{1,2} \\ V_{3'}^{1,2} \end{bmatrix} = 0 \qquad (4\text{-}7)$$

where $U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

or:

$$\begin{bmatrix} 0 \\ V_{1'}^{1,2} \\ V_{2'}^{1,2} \\ V_{3'}^{1,2} \end{bmatrix} = - \begin{bmatrix} U & -U & U & -U \\ 0 & 0 & U & 0 \\ U & 0 & U & 0 \\ U & -U & U & 0 \end{bmatrix} \begin{bmatrix} V_a^{1,2} \\ V_b^{1,2} \\ V_c^{1,2} \\ V_d^{1,2} \end{bmatrix} + \begin{bmatrix} U & 0 \\ U & 0 \\ U & 0 \\ U & 0 \end{bmatrix} \begin{bmatrix} V_{01}^{1,2} \\ V_{03}^{1,2} \end{bmatrix} \qquad (4\text{-}8)$$

Substituting the terminal equations of equation (4-6) at positive and negative sequence

and $$V_c^{1,2} = Z_{cc}^{1,2}\; I_{cc}^{1,2} \qquad \text{(generator B)}$$

and $$V_d^{1,2} = Z_{dd}^{1,2}\; I_{dd}^{1,2} \qquad \text{(generator A)}$$

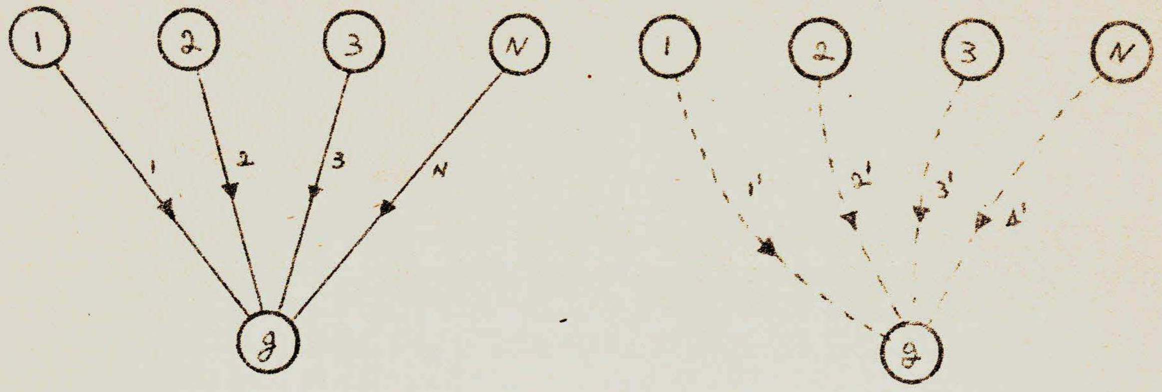
into the equation (4-8), and expressing the brench currents in terms of chord currents we have:

$$\begin{bmatrix} 0 \\ V_{1'}^{1,2} \\ V_{2'}^{1,2} \\ V_{3'}^{1,2} \end{bmatrix} = - \begin{bmatrix} U & -U & U & -U \\ 0 & 0 & U & 0 \\ U & 0 & U & 0 \\ U & -U & U & 0 \end{bmatrix} \begin{bmatrix} Z_{aa}^{1,2} & Z_{ab}^{1,2} & 0 & 0 \\ Z_{ab}^{1,2} & Z_{bb}^{1,2} & 0 & 0 \\ 0 & 0 & Z_{cc}^{1,2} & 0 \\ 0 & 0 & 0 & Z_{dd}^{1,2} \end{bmatrix} \begin{bmatrix} U & 0 & U & U \\ -U & 0 & 0 & -U \\ U & U & U & U \\ -U & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{03}^{1,2} \\ I_{1'}^{1,2} \\ I_{2'}^{1,2} \\ I_{3'}^{1,2} \end{bmatrix} + \begin{bmatrix} U & 0 \\ U & 0 \\ U & 0 \\ U & 0 \end{bmatrix} \begin{bmatrix} V_{01}^{1,2} \\ V_{03}^{1,2} \end{bmatrix} \qquad (4\text{-}9)$$

From the top equation of the equation (4-9) we can determine the value of $I_{03}^{1,2}$ and substitute it into the rest of equation (4-9), then we have a relation between:

$$\begin{bmatrix} V_{1'}^{1,2} \\ V_{2'}^{1,2} \\ V_{3'}^{1,2} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} I_{1'}^{1,2} \\ I_{2'}^{1,2} \\ I_{3'}^{1,2} \end{bmatrix}$$

as:



a)

b)

Figure 4-6

a) Terminal graph.

b) Augmented source drivers.

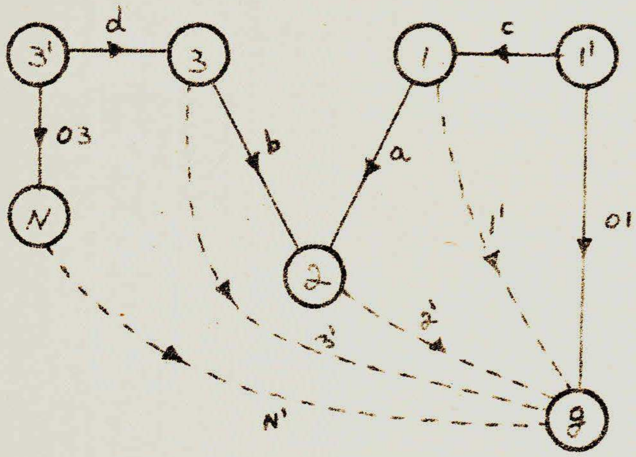


Figure 4-7

Overall graph.

$$\begin{array}{c|cccccccccc|} & a & b & c & d & 01 & 03 & 1' & 2' & 3' & N' \\ \hline 1' & 1 & -1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 2' & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 3' & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ N' & 1 & -1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{array} \begin{bmatrix} V_a^0 \\ V_b^0 \\ V_c^0 \\ V_d^0 \\ V_{01}^0 \\ V_{03}^0 \\ V_{1'}^0 \\ V_{2'}^0 \\ V_{3'}^0 \\ V_{N'}^0 \end{bmatrix} = 0 \qquad (4\text{-}11)$$

where the elements (a, b, c, d, 01 and 03) constitute the tree.

With $V^0_{01} = V^0_{03} = 0$ the equation (4-11) is:

$$\begin{bmatrix} V^0_{1'} \\ V^0_{2'} \\ V^0_{3'} \\ V^0_{N'} \end{bmatrix} = - \begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} V^0_a \\ V^0_b \\ V^0_c \\ V^0_d \end{bmatrix} \qquad (4\text{-}12)$$

Substituting the terminal equation, equation (4-6), at zero sequence and

$$V^0_c = Z^0_{cc} \, I^0_c \qquad \text{(generator B)}$$

$$V^0_d = V^0_{dd} \, I^0_d \qquad \text{(generator A)}$$

into equation (4-12) and then expressing the brench currents in terms of chord currents we have:

$$\begin{bmatrix} V^0_{1'} \\ V^0_{2'} \\ V^0_{3'} \\ V^0_{N'} \end{bmatrix} = - \begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} Z^0_{aa} & Z^0_{ab} & 0 & 0 \\ Z^0_{ab} & Z^0_{bb} & 0 & 0 \\ 0 & 0 & Z^0_{cc} & 0 \\ 0 & 0 & 0 & Z^0_{dd} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I^0_{1'} \\ I^0_{2'} \\ I^0_{3'} \\ I^0_{N'} \end{bmatrix} \qquad (4\text{-}13)$$

as:

$$\begin{bmatrix} V^0_{1'} \\ V^0_{2'} \\ V^0_{3'} \\ V^0_{N'} \end{bmatrix} = \begin{bmatrix} V^0_{1} \\ V^0_{2} \\ V^0_{3} \\ V^0_{N} \end{bmatrix} = V^0_{BUS} \quad \text{and} \quad - \begin{bmatrix} I^0_{1'} \\ I^0_{2'} \\ I^0_{3'} \\ I^0_{N'} \end{bmatrix} = \begin{bmatrix} I^0_{1} \\ I^0_{2} \\ I^0_{3} \\ I^0_{N} \end{bmatrix} = I^0_{BUS}$$

we have $V^0_{BUS} = Z^0_{BUS}\ I^0_{BUS}$ (4-14a)

$$\begin{bmatrix} V^0_1 \\ V^0_2 \\ V^0_3 \\ V^0_N \end{bmatrix} = \begin{bmatrix} 0.02 & 0.02 & 0.02 & 0.02 \\ 0.02 & 0.0664 & 0.0454 & 0.0454 \\ 0.02 & 0.0454 & 0.1107 & 0.1107 \\ 0.02 & 0.0453 & 0.1107 & 0.1407 \end{bmatrix} \begin{bmatrix} I^0_1 \\ I^0_2 \\ I^0_3 \\ I^0_N \end{bmatrix} \qquad (4\text{-}14b)$$

From the results of positive, negative and zero sequence we can write:

$V^{0,1,2}_{BUS} = Z^{0,1,2}_{BUS}\ I^{0,1,2}_{BUS} + V^{0,1,2}_{0}$ (4-15a)

$$\begin{bmatrix} V^{0,1,2}_1 \\ V^{0,1,2}_2 \\ V^{0,1,2}_3 \\ V^{0,1,2}_N \end{bmatrix} \begin{bmatrix} \\ \\ Z^{0,1,2}_{BUS} \\ \\ \end{bmatrix} \begin{bmatrix} I^{0,1,2}_1 \\ I^{0,1,2}_2 \\ I^{0,1,2}_3 \\ I^{0,1,2}_N \end{bmatrix} + \begin{bmatrix} 0 \\ 1.82 + j0 \\ 0 \\ 0 \\ 1.82 + j0 \\ 0 \\ 0 \\ 1.82 + j0 \\ 0 \\ 0 \\ 1.82 + j0 \\ 0 \end{bmatrix} \qquad (4\text{-}15b)$$

From the graph of the faulted system at bus (2) Figure 4-8 we have:

$$I_F^{0,1,2} = - I_2^{0,1,2}$$

$$I_1^{0,1,2} = I_3^{0,1,2} = I_N^{0,1,2} = 0$$

$$V_F^{0,1,2} = V_2^{0,1,2} = Z_F^{0,1,2} \; I_F^{0,1,2}$$

where $Z_F^{0,1,2} = (Y_F^{0,1,2})^{-1}$

Substituting these values into equation (4-15) we have:

$$I_F = Y_F^{0,1,2} \; [U + Y_F^{0,1,2} \; Z_{22}^{0,1,2}]^{-1} \; V_0^{0,1,2}$$

at $Y_F^{0,1,2} = y_f/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ where $y_f = (z_f)^{-1}$.

we have:

$$I_F^{0,1,2} = \frac{\sqrt{3} \times 1.05}{Z_{22}^0 + Z_{22}^1 + Z_{22}^2 + 3z_f} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$I_F^{a,b,c} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} I_F^{0,1,2}$$

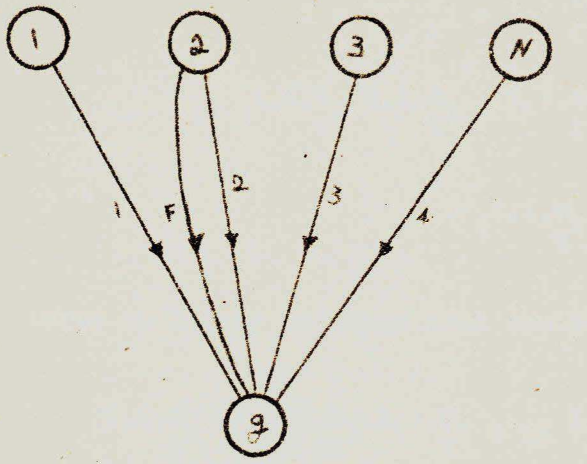


Figure 4-8

Equivalent graph of the faulted system.

$$I_F^{a,b,c} = \frac{3 \times 1.05}{Z_{22}^{(0)} + Z_{22}^{(1)} + Z_{22}^{(2)} + 3z_f} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

at $z_f = 0$

$$I_F^{a,b,c} = \begin{bmatrix} \dfrac{3.15}{j0.0664 + j0.1345 + j0.0899} \\ 0 \\ 0 \end{bmatrix}$$

$$I_F^{a,b,c} = \begin{bmatrix} -j10.86 \\ 0 \\ 0 \end{bmatrix}$$

$$V_F^{0,1,2} = \frac{\sqrt{3} \times 1.05 \ =}{Z_{22}^{0} + Z_{22}^{1} + Z_{22}^{2} + 3z_f} \begin{bmatrix} -\ Z_{22}^{0} \\ Z_{22}^{0} + Z_{22}^{1} + 3z_f \\ -\ Z_{22}^{1} \end{bmatrix}$$

$$V_F^{a,b,c} = \begin{bmatrix} \dfrac{3z_f}{Z_{22}^{0} + Z_{22}^{1} + Z_{22}^{2} + 3z_f} \\ a^2 - \dfrac{Z_{22}^{0} - Z_{22}^{1}}{Z_{22}^{0} + Z_{22}^{1} + Z_{22}^{2} + 3z_f} \\ a - \dfrac{Z_{22}^{0} - Z_{22}^{1}}{Z_{22}^{0} + Z_{22}^{1} + Z_{22}^{2} + 3z_f} \end{bmatrix} 1.05$$

at $z_f = 0$

$$V_F^{a,b,c} = \begin{bmatrix} 0 \\ -\ 0.5 - j0.084 \\ -\ 0.5 + j0.084 \end{bmatrix}$$

$$V_N^0 = Z_{22}^0\ I_F^0$$

$$V_N = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_N^0 \\ 0 \\ 0 \end{bmatrix} = V_N = \frac{V_N^0}{\sqrt{3}} = 0.16$$

5. TRANSIENT STABILITY SIMULATION

This simulation will be done for a particular power system given in the paper, "TRANSIENT STABILITY REGIONS OF MULTI-MACHINE POWER SYSTEMS by A. H. EL-ABIAD and K. NAGAPPAN. The system has four generators and seven lines. Figure 5-1 shows the single line diagram of the system.

System data (100 MVA base)

Line		pu Impedances		Line Charging
From Bus	To Bus	R	X	Y/2
1	2	0.05	0.20	0
1	6	0.10	0.15	0
2	3	0.10	0.50	0
2	3	0.20	0.50	0
3	4	0.20	0.80	0
4	5	0.10	0.30	0
5	6	0.20	0.40	0

OBSERVATION

In this work the values of Y/2 will be neglected.

	Loads	
At Bus	P	Q
2	0.20	0.10
5	0.40	0.15
6	0.30	0.10

Synchronous machine constants

Generator Number	At Bus	Capacity	M	X_d
10	1	1.0	0.53017	j0.004
7	2	0.15	0.00795	j1.0
8	3	0.40	0.01590	j0.5
9	4	0.30	0.01060	j0.4

Load flow for prefault condition (excluding machine reactances)

Bus	E	Degrees	P_G	Q_G	P_L	Q_L
1	1.000	0.0	0.332	0.091	0.0	0.0
2	1.002	-0.12	0.10	0.05	0.20	0.10
3	1.084	4.62	0.30	0.20	0.0	0.0
4	1.025	1.41	0.20	0.10	0.0	0.0
5	0.956	-2.80	0.0	0.0	0.40	0.15
6	0.953	-2.30	0.0	0.0	0.30	0.10

Loss for P_L = 0.032

Loss for Q_L = 0.090

P_m Input

Generator Number	P_m
10	0.332
7	0.10
8	0.30
9	0.20

CONSIDERATIONS

1) Saliency and saturation will be neglected for all machines.

2) Each machine will be represented by a constant reactance (direct axis transient reactance) in series with a constant e.m.f. (voltage behind transient reactance; constant flux linkages). Stator resistance of the machines will be neglected.

3) The mechanical input power to all machines will remain constant during the transient period.

4) Load will be converted into equivalent admittance bases on voltage values obtained from load flow solution.

5) The disturbance on the system will be a balanced fault bus to ground, with only one switching operation.

Swing equation

$$M \frac{d^2\delta}{dt^2} = P_m - P_e \qquad (5\text{-}1)$$

where M = inertia constant in p.u.

$$M = \frac{H}{\pi f}$$

with H = MW. sec. MVA^{-1}.

p_m = mechanical power in p.u.

P_e = electrical power in p.u.

δ = rotor electrical angle in radians with respect to a synchronously rotating reference frame.

A consequence of the synchronous machine modelling is that δ as defined above is the same as the phase angle associated with the voltage being transient reactance.

Used Method

The terminal representation at the buses 1', 2', 3' and 4' of the generators is obtained for the faulted and post faulted states (1', 2', 3' and 4' are internal buses of the generators).

Considering the terminal representation of the transmission system, Figure 5-2, where the corresponding admittance matrix is $Y_{BL} = Y_{BUS}$ + the elements corresponding to the load admittance,which may be Y_{BLF} or Y_{BLPF}, at faulted and post faulted states respectively; but are assumed to be identical in our development whose elements are represented by Y_{ij}, augmenting the terminal graphs of Figure 5-2 by elements corresponding to generator reactances as shown in Figure 5-3.

Now, eliminating the nodes 1 to 6 we can determine a terminal representation of Figure 5-4, where the corresponding admittance matrix is Y_{IB} (which may be Y_{IBF} or Y_{IBPF},at faulted and post faulted states respectively).

The voltages and currents associated with the terminal graph of Figure 5-4 are:

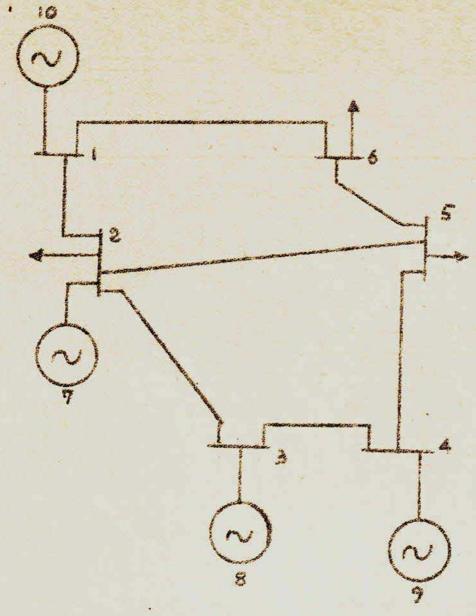


Figure 5-1

Single line diagram of the system.

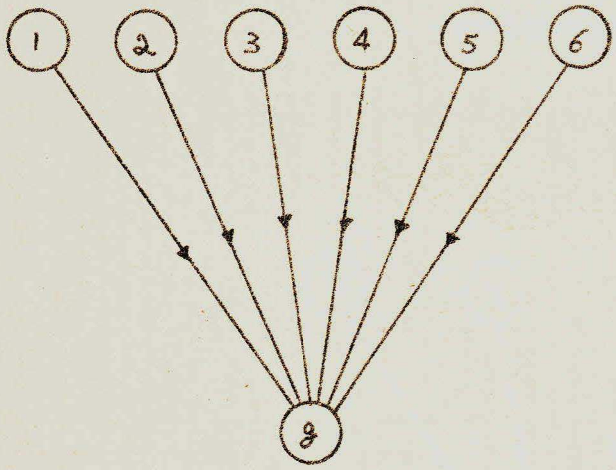


Figure 5-2

Terminal representation of the transmission system.

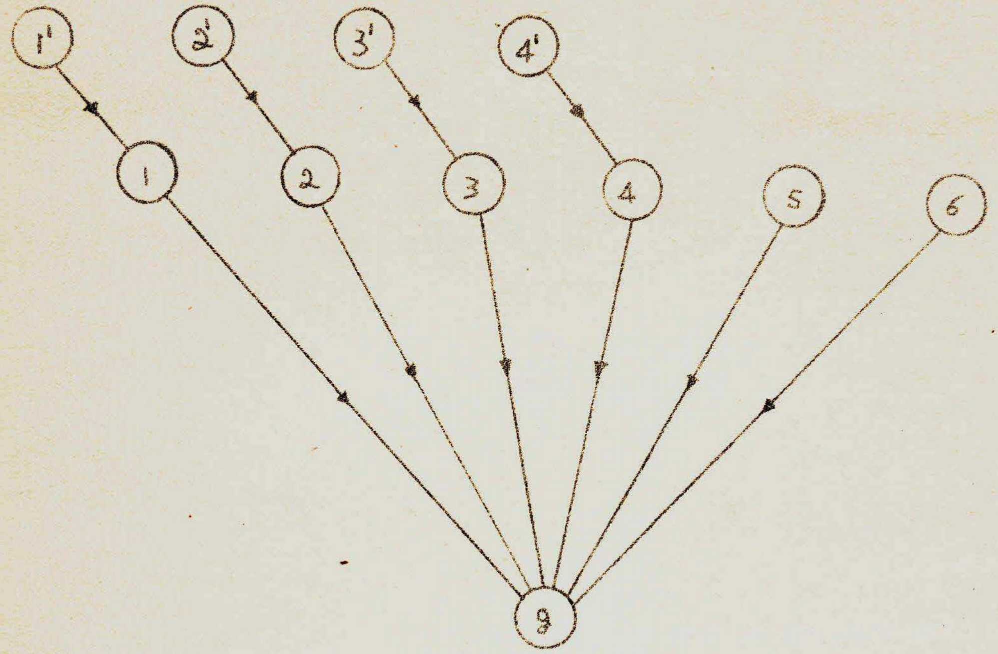


Figure 5-3

Terminal graph of the Figure 5-2 augmented by elements corresponding to generator reactances.

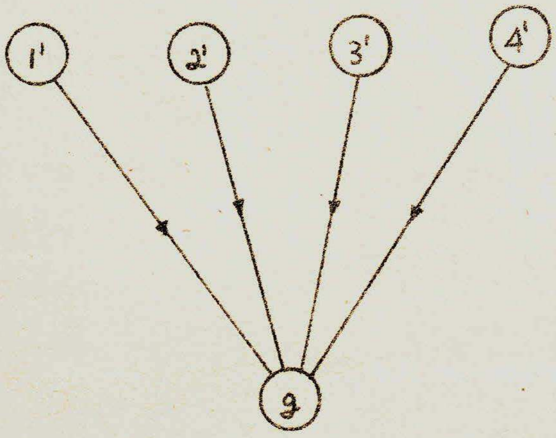


Figure 5-4

Terminal graph corresponding to internal buses of the generators.

$$V' = \begin{bmatrix} V'_1 \\ V'_2 \\ V'_3 \\ V'_4 \end{bmatrix} \quad \text{and } I' = \begin{bmatrix} I'_1 \\ I'_2 \\ I'_3 \\ I'_4 \end{bmatrix}$$

The electric power delivered by $i^{\underline{th}}$ machine is given as:

$$P_{ei} = R_e \, [V'^{*}_i \; I'_i] \tag{5-2}$$

$$P_{ei} = R_e \, [V'^{*}_i \sum_{j=1}^{4} Y_{ij} \, V'_j]$$

At $\quad Y_{ij} = G_{ij} - jB_{ij} = |Y_{ij}| \angle \theta_{ij}$

$$V'^{*}_i = |V'_i| \angle -\delta_i \quad \text{and} \quad V'_j = |V'_j| \angle \delta_j$$

we have:

$$P_{ei} = |V'_i|^2 \, G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{4} |V'_i| \, |V'_j| \, |Y_{ij}| \cos [\theta_{ij} - (\delta_i - \delta_j)] \tag{5-3}$$

where $|V'_1|$, $|V'_2|$, $|V'_3|$ and $|V'_4|$ will be held constants at the values calculated from load flow.

For appropriate Y_{ij} and P_{ei} given as equation (5-3) the equation (5-1) becomes:

at $0 < t \leq T_c$ (faulted states)

$$P_{mi} = P_{ei}^{F} + M_i \frac{d^2\delta_i}{dt^2} \qquad (5\text{-}4)$$

and

at $t \geq T_c$ (post faulted states)

$$P_{mi} = P_{ei}^{PF} + M_i \frac{d^2\delta_i}{dt^2} \qquad (5\text{-}5)$$

with $\delta_i\Big|_{t=0} = \delta_i^{(0)} \quad \frac{d\delta_i}{dt}\Big|_{t=0} = 0$

where T_c = Time which the fault is cleared after one switching operation.

Now, using a program in WATFIV developed by Paul Kish we can determine the swing curve with $T_c = 0.42s$ and $T_c = 0.44s$, when a three phase to ground fault occurs at the bus number 3.

Program output

Y_{ij} (faulted states)

$$\begin{bmatrix} 1.01 - j2.72 & -0.01 + j0.58 & 0.0 + j0.0 & -0.08 + j0.55 \\ -0.01 + j0.58 & 0.03 - j0.89 & 0.0 + j0.0 & 0.01 + j0.04 \\ 0.0 + j0.0 & 0.0 + j0.0 & 0.0 - j2.00 & 0.0 + j0.0 \\ -0.08 + j0.55 & 0.01 + j0.04 & 0.0 + j0.0 & 0.22 - j1.34 \end{bmatrix}$$

Y_{ij} (post faulted states)

$$\begin{bmatrix} 0.85 - j2.30 & -0.01 + j0.66 & -0.08 + j0.64 & -0.11 + j0.76 \\ -0.01 + j0.66 & 0.03 - j0.88 & 0.02 + j0.11 & 0.01 + j0.08 \\ -0.08 + j0.64 & 0.02 + j0.11 & 0.11 - j1.09 & 0.02 + j0.30 \\ -0.11 + j0.76 & 0.01 + j0.08 & 0.02 + j0.30 & 0.22 - j1.24 \end{bmatrix}$$

The Figures 5-5 and 5-6 show the swing curves for the system at $T_c = 0.42s$ and $T_c = 0.44s$ respectively.

We have at $T_c = 0.44s$ unstable for the generator at bus number three.

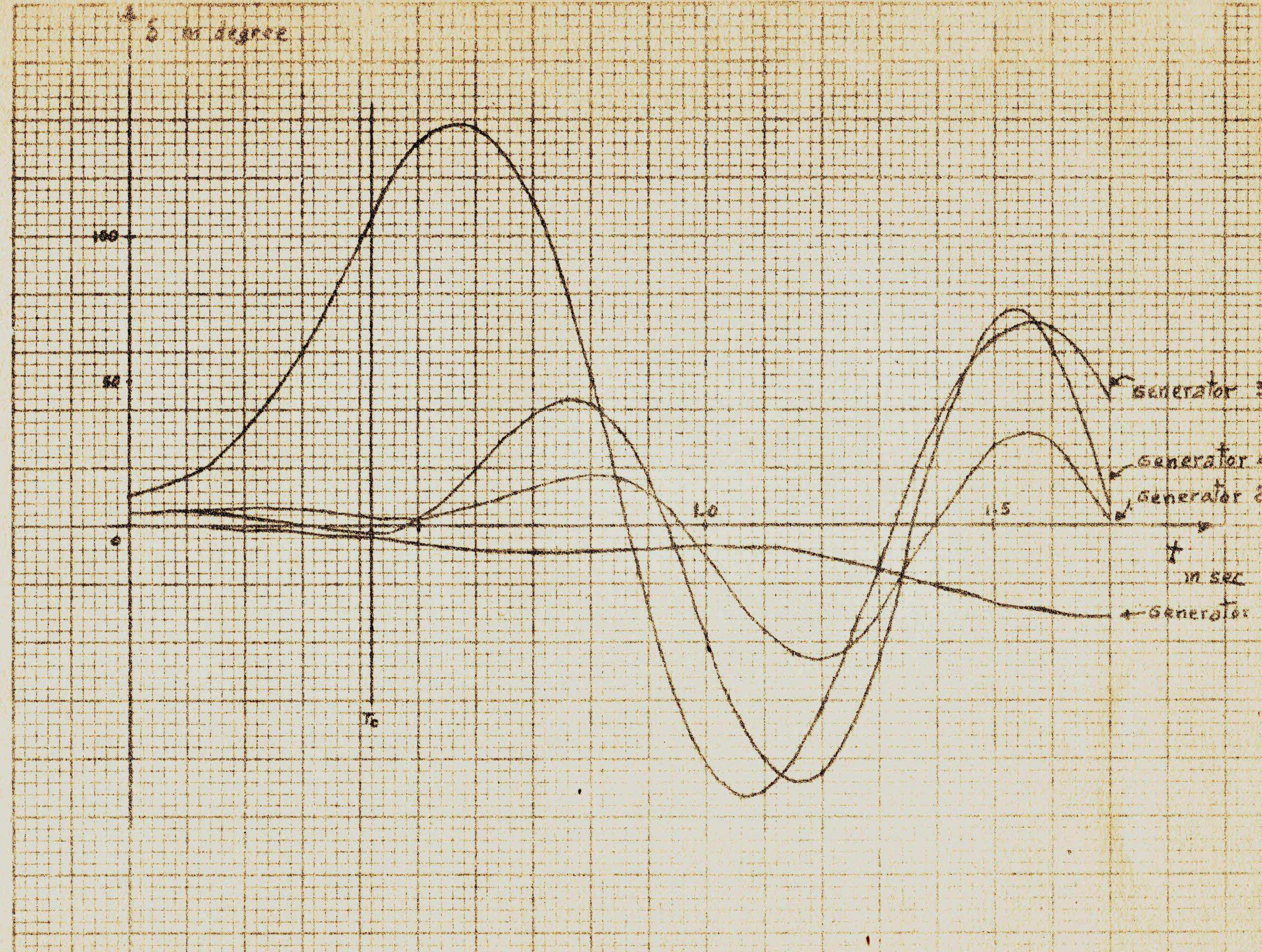
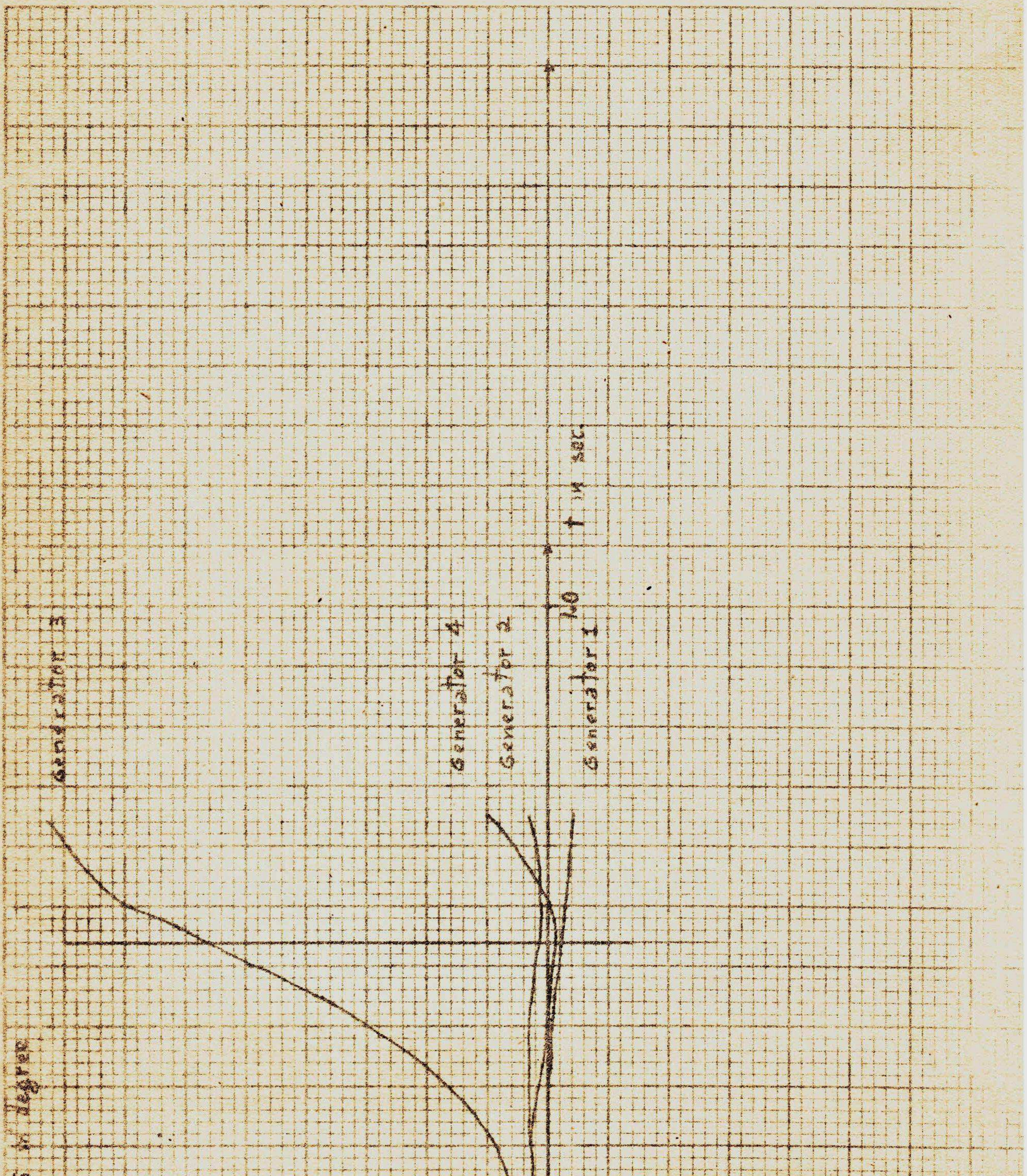

δ in degree
100
50
0
T_c
1.0
1.5
t in sec
Generator 3
Generator 4
Generator 2
Generator 1

fig. 5-5


δ in degree
150
100
50
20
t in sec
Generator 3
Generator 4
Generator 2
Generator 1

CONCLUSION

In this work we have simulated short circuit, load flow and transient stability studies, using a particular power system; but these simulations can be extended for other systems and for different kinds of studies, for example; economical operation, system planning, analysis of further modifications of the system, etc.

Therefore, the purpose of this work was to show the graph theory application for power system analysis.

BIBLIOGRAPHY

1. H. K. Kesavan and M. A. Pai, "Graph-Theoretic Models in Power System Analysis", July, 1971.
2. Glenn W. Stagg and Ahmed H. El-Abiad, "Computer Methods in Power System Analysis", McGraw-Hill, 1968.
3. E. W. Kimbard, "Power System Stability", 3 vol., 1950.
4. W. D. Stevenson, "Elements of Power System Analysis".
5. B. M. Weedy, "Electric Power Systems", John Wiley, 1967.
6. A. H. El-Abiad and K. Nagappan, "Regions of Multimachine Power System", IEEE Trans. Power Apparatus and System, February, 1966.

APPENDIX

```
-BUG***** UNIVERSITY OF WATERLOO *****14.59.52****FRIDAY    **** 3 DEC 71****
 $JOB     WATFIV  T0628KES.CHAND,KP=29
 C        ************************************************************************
 C         Z BUS AND SHORT CIRCUIT PROGRAM
 C        ************************************************************************
 2        FORMAT(72A1)
 30       FORMAT(1X,72A1)
 121      FORMAT(10X,I2)
 122      FORMAT(10X,I2,1X,I2,1X,4F10.4)
 123      FORMAT(1X,'BRANCH',4X,2(I2,1X),4F10.4)
 124      FORMAT(1X,'CHORD ',4X,2(I2,1X),4F10.4)
 125      FORMAT(1X,'REFNC.',4X,I2)
 444      FORMAT(1X,1X,I2,2F10.4)
 402      FORMAT(12F10.4)
 606      FORMAT(1H1)
 5001     FORMAT(' ***************************************************************')
 5002     FORMAT(/)
 5003     FORMAT(' ERROR - ILLEGAL CODE',A1)
 5004     FORMAT(' INITIAL ZBUS---EO,  ORDER=',I2)
 5005     FORMAT(20F10.4)
 5007     FORMAT('                LINE SPECIFICATION')
 5008     FORMAT('  CODE        NODES                 Z-VALUE                 EO-VALUE')
 5009     FORMAT(' ZBUS')
 5010     FORMAT(' EO')
 5011     FORMAT(' FAULT ANALYSIS')
 5012     FORMAT(' BUS MAGNITUDE      ANGLE')
          DIMENSIONF1(20),F2(20) COMPLEXF COMPLEXZ(21,21),EO(21),ZDIF(21),ZP
         *(21),ZVAL,EVAL,ZHAT,YHAT,ZPP,EOP,EDIF INTEGERICH(72),IC(6) DATAICH
         *R/99/ DATAIC/'Z',' ',' ','P','R','E'/ PRINT606 1234 CONTINUE
          READ2,ICH PRINT5001 PRINT30,ICH PRINT5001 PRINT5002
          DO1I=1,21 Z(21,I)=0. 1 Z(I,21)=0. EO(21)=0. NREF=21 1000 CONTINUE
          READ2,ICH DO3I=1,6 3 IF(ICH(1).EQ.IC(I))GOTO4 PRINT5003,ICH(1) GOT
         *01000 4 WRITE(ICHR,2)ICH GOTO(100,222,222,400,500,600),I
 100      READ(ICHR,121)N PRINT5004,N
          DO101I=1,N READ5005,(Z(I,J),J=1,N),EO(I) 101 PRINT402,(Z(I,J),J=1,
         *N),EO(I) PRINT5002 PRINT5007 PRINT5008 PRINT5002 GOTO1000
 222      READ(ICHR,122)NF,NT,ZVAL,EVAL IF(NF.GT.N.OR.NT.GT.N)GOTO200 GOTO30
         *0
 200      CONTINUE PRINT123,NF,NT,ZVAL,EVAL
          IF(NF.LT.NT)EVAL=-EVAL IF(NT.EQ.0)NT=NREF IF(NF.EQ.0)NF=NREF I=NT
          NT=NF NF=I DO201I=1,N Z(NF,I)=Z(NT,I) 201 Z(I,NF)=Z(I,NT) Z(NF,NF)
         *=Z(NT,NT)+ZVAL EO(NF)=EO(NT)+EVAL N=NF GOTO1000
 300      CONTINUE PRINT124,NF,NT,ZVAL,EVAL IF(NF.EQ.0)NF=NREF IF(NT.EQ.0)NT
         *=NREF ZHAT=Z(NF,NF)-2.*Z(NF,NT)+Z(NT,NT)+ZVAL YHAT=1./ZHAT DO301I=
         *1,N 301 ZDIF(I)=Z(I,NF)-Z(I,NT) EDIF=(EO(NF)-EO(NT)-EVAL)*YHAT DO30
         *2I=1,N DO303J=1,N 303 Z(I,J)=Z(I,J)-YHAT*ZDIF(I)*ZDIF(J) 302 EO(I
         *)=EO(I)-EDIF*ZDIF(I) GOTO1000
 400      PRINT5002 PRINT5009 PRINT5002
          DO401I=1,N F=EO(I)/Z(I,I) F1(I)=CABS(F) F2(I)=0. IF(REAL(F).GT.0.)
         *F2(I)=ATAN(AIMAG(F)/REAL(F))*180./(4.*ATAN(1.)) 401 PRINT402,(Z(I,
         *J),J=1,N) PRINT5002 PRINT5010 PRINT5002 DO404I=1,N 404 PRINT402,EO
         *(I)
          PRINT5002 PRINT5011 PRINT5012 DO445I=1,N 445 PRINT444,I,F1(I),F2(I
         *) GOTO1000
 500      CONTINUE READ(ICHR,122)NB PRINT125,NB DO503I=1,N 503 ZP(I)=Z(I,NB)
          ZPP=Z(NB,NB) EOP=EO(NB) DO501I=1,N DO501J=1,N 501 Z(I,J)=Z(I,J)+ZP
         *P-ZP(J)-ZP(I) DO502I=1,N EO(I)=EO(I)-EOP Z(NB,I)=Z(NB,I)-ZP(I)+ZPP
 502      Z(I,NB)=Z(I,NB)-ZP(I)+ZPP Z(NB,NB)=Z(NB,NB)+ZPP EO(NB)=EO(NB)-EOP
          GOTO1000
 600      PRINT5002 GOTO1234 END

 $ENTRY
```

```
BUG**** UNIVERSITY OF WATERLOO ****14.31.47****FRIDAY    **** 3 DEC 71**
$JOB    WATFIV  P1023NRC.KESAVAN,KP=29
C      ****************************************************************
C       LOAD FLOW PROGRAM - YBUS -NEWTON-RAPHSON .
C      ****************************************************************
C NBUS IS THE TOTAL NUMBER OF BUSES TO BE GIVEN
C NLIN IS THE TOTAL NUMBER OF LINES THAT MUST BE SPECIFIED
C I  THE TOTAL ADMITTANCE YG ALONE IS GIVEN WITHOUT DISTRIBUTING IT TO
C EACH END OF A LINE THEN INDEX=1
C IBFR IS THE VECTOR CONTAINING THE NUMBER OF THE BUS FROM WHICH A LINE
C STARTS.
C IBTO IS A VECTOR HOLDING THE NUMBER OF THE BUS AT WHICH THE LINE  TERM
C IBTO IS A VECTOR HOLDING THE NUMBER OF THE BUS AT WHICH THE LINE  TERM
C IBTO IS A VECTOR HOLDING THE NUMBER OF THE BUS AT WHICH THE LINE  TERM
C LINN IS THE VECTOR CONTAINING THE ORDER OF THE LINES.
C BIMP IS THE BUS IMPEDENCE VECTOR.
C  BUS IS THE BUS ADMITTANCE  VECTOR.
C BUSA IS THE STATIC CAPACITOR VECTOR.
C YBUS IS THE FINAL BUS ADMITTANCE VECTOR.
      DIMENSIONLINN(25),IBFR(25),IBTO(25),IDENA(25),IDENB(25),STEP(20),S
     *TART(20),SHIFT(20),VOLMX(20),VOLMI(20),PLD(20),QLD(20),PGEN(20),QG
     *EN(20),G(20,20),B(20,20),ERAI(40),CUR(40),ABSOL(40),GIVEA(40) COMP
     *LEXBIMP(25),BUS(25),BUSA(20),YBUS(20,20),ZA,YG,CYBUS(20,20),YA,YB,
     *VOLT(20),SUM,YAB1,YAB2 REAL  JMTRX(40,40),GIVEN(40) DIMENSIONIR(40
     *),IC(40) COMPLEXCMPLX,CONJG DIMENSIONALPA(2,30) 810 FORMAT(2X,'BAS
     *E ET',F10.6,11I10)
    2 FORMAT(2I5,4F10.0,I5)
   30 FORMAT(2X,'I',I2,'J',I2,'YBUS',2F15.5,'CYBUS',2F15.5/(3X,I2,1X,I2,
     *6X,2F15.5,7X,2F15.5))
  31  FORMAT(F10.0,11I5)
   32 FORMAT(I5,2F10.4,2F10.4,2F5.3,2F10.4,I5)
   35 FORMAT(2I5,5F10.4)
 760   FORMAT(10(2A4))
      LRD=5 ITERAT=1 LWE=6 READ(LRD,31)BASE,NLIN,NBUS,NGEN,INDEX,NTR,NFP
     *AJ,NFPYB,NFPDI,NFPBI,NFPLI,IGAN NFPCR=NFPAJ NFPVO=NFPAJ NFPGI=NFPA
     *J IF(NFPDI.NE.1)GOTO201 WRITE(LWE,810)BASE,NLIN,NBUS,NGEN,INDEX,NT
     *R,NFPAJ,NFPYB,NFPDI,NFPBI,NFPLI,IGAN 201 DO160M=1,NBUS DO160N=1,NB
     *US 160 YBUS(M,N)=CMPLX(0.,0.) READ(LRD,760)((ALPA(I,J),I=1,2),J=1,
     *NBUS) READ(LRD,32)(N,PLD(N),QLD(N),BUSA(N),VOLT(N),PGEN(N),QGEN(N)
     *,IDENB(N),M=1,NBUS) IF(NFPBI.NE.1)GOTO202 WRITE(LWE,811)(N,PLD(N),
     *QLD(N),BUSA(N),VOLT(N),PGEN(N),QGEN(N),IDENB(N),N=1,NBUS) 811 FORM
     *AT(2X,'N,PLD ET',I10,8F12.5,I5/(10X,I10,8F12.5,I5))
 202  ANJA=1. DO1L=1,NLIN READ(LRD,2)LA,MA,ZA,YG,I IDENA(L)=I LINN(L)=L
      BIMP(L)=ZA IBFR(L)=LA IBTO(L)=MA BUS(L)=YG 1 CONTINUE IF(NFPLI.NE.
     *1)GOTO207 WRITE(LWE,208)(IBFR(L),IBTO(L),BIMP(L),BUS(L),IDENA(L),L
     *=1,NLIN) 208 FORMAT(2X,'LINE DATA',2I5,4F10.5,I5/(11X,2I5,4F10.5,I
     *5))
C CALCULATE BUS SHUNT ADMITTANCE
 207  DO3J=1,NBUS SUM=CMPLX(0.,0.) DO4I=1,NLIN IF(IBFR(I).EQ.J)GOTO5 IF(
     *IBTO(I).EQ.J)GOTO5 GOTO4 5 IF(INDEX.EQ.1)GOTO6 SUM=SUM+BUS(I) GOTO
     *4 6 SUM=SUN+BUS(I)/2. 4 CONTINUE BUSA(J)=SUM+BUSA(J) 3 CONTINUE DO
     *730I=1,NLIN IF(INDEX.NE.1)GOTO730 BUS(I)=BUS(I)/2. 730 CONTINUE
C CALCULATE LINE ADMITTANCE
      DO9I=1,NLIN BIMP(I)=1./BIMP(I) 9 CONTINUE
C CALCULATE YBUS ADMITTANCE
      DO10J=1,NBUS SUM=CMPLX(0.,0.) DO11I=1,NLIN IF(IBFR(I).EQ.J)GOTO12
      IF(IBTO(I).EQ.J)GOTO13 GOTO11 12 SUM=SUM+BIMP(I) L=IBTO(I) LN=J GO
     *TO14 13 SUM=SUM+BIMP(I) L=IBFR(I) LN=J 14 YBUS(LN,L)=-BIMP(I)+YBUS
     *(LN,L) 11 CONTINUE YBUS(J,J)=SUM+BUSA(J) 10 CONTINUE DO33I=1,NBUS
      DO33J=1,NBUS 33 CYBUS(I,J)=YBUS(I,J) IF(NFPYB.NE.1)GOTO203 WRITE(L
     *WE,30)((I,J,YBUS(I,J),CYBUS(I,J),J=1,NBUS),I=1,NBUS) 203 IF(IGAN.E
     *Q.7)GOTO607
```

```
C INPUT PARAMETERS OF THE OFF-LOAD TAP CHANGE TRANSFORMERS.
      DO34I=1,NTR READ(LRD,35)LA,MA,ST,STA,SHI,VOLM,VOLE DO36J=1,NLIN IF
     *(IBFR(J).EQ.LA.AND.IBTO(J).EQ.MA)GOTO37 36 CONTINUE 37 STEP(J)=ST
      START(J)=STA SHIFT(J)=SHI VOLMX(J)=VOLM VOLMI(J)=VOLE 34 CONTINUE
C CALCULATION OF YBUS WITH OFF-LOAD TAP CHANGING TRANSFORMERS.
      DO44L=1,NLIN N=IBTO(L) IF(IDENA(L).EQ.1)GOTO38 GOTO44 38 IF(VOLMX(
     *L).LT.CABS(VOLT(N)))GOTO39 GOTO40 39 START(L)=START(L)+STEP(L)
   40 IF(VOLMI(L).GT.CABS(VOLT(N)))GOTO41 GOTO42 41 START(L)=START(L)-ST
     *EP(L) 42 LN=IBFR(L) LM=IBTO(L) ANJ=1./START(L) ANJB=(ANJ**2)-(ANJA
     ***2) YBUS(LN,LN)=YBUS(LN,LN)-(ANJB*YBUS(LN,LM)) ANJB=ANJ-ANJA YBUS
     *(LN,LM)=(YBUS(LN,LM)*ANJB)+(YBUS(LN,LM)) YBUS(LM,LN)=YBUS(LN,LM)
   44 CONTINUE IF(NFPYB.NE.1)GOTO607 WRITE(LWE,30)((I,J,YBUS(I,J),CYBUS(
     *I,J),J=1,NBUS),I=1,NBUS)
C CHANGE ROWS AND COLUMNS SO THAT THE SWING BUS IS PLACED IN THE LAST
C ROW AND LAST COLUMN.
  607 DO71I=1,NBUS IF(IDENB(I).NE.3)GOTO71 LA=I LOVE=LA LB=NBUS 71 CONTI
     *NUE DO81I=1,NBUS IF(I.NE.LA)GOTO81 DO750J=1,NBUS SUM=YBUS(I,J) YBU
     *S(I,J)=YBUS(LB,J) YBUS(LB,J)=SUM 750 CONTINUE 81 CONTINUE DO82I=1,
     *NBUS IF(I.NE.LA)GOTO82 DO751J=1,NBUS SUM=YBUS(J,I) YBUS(J,I)=YBUS(
     *J,LB) YBUS(J,LB)=SUM 751 CONTINUE 82 CONTINUE IF(NFPYB.NE.1)GOTO20
     *4 WRITE(LWE,30)((I,J,YBUS(I,J),CYBUS(I,J),J=1,NBUS),I=1,NBUS)
C SHIFT SWING BUS PARAMETERS TO THE LAST ROW.
204   DO70N=1,NBUS IF(IDENB(N).NE.3)GOTO70 TEMP=PLD(N) PLD(N)=PLD(NBUS)
      PLD(NBUS)=TEMP TEMP=QLD(N) QLD(N)=QLD(NBUS) QLD(NBUS)=TEMP YA=BUSA
     *(N) BUSA(N)=BUSA(NBUS) BUSA(NBUS)=YA YA=VOLT(N) VOLT(N)=VOLT(NBUS)
      VOLT(NBUS)=YA TEMP=PGEN(N) PGEN(N)=PGEN(NBUS) PGEN(NBUS)=TEMP TEMP
     *=QGEN(N) QGEN(N)=QGEN(NBUS) QGEN(NBUS)=TEMP NTEMP=IDENB(N) IDENB(N
     *)=IDENB(NBUS) IDENB(NBUS)=NTEMP 70 CONTINUE NBU=NBUS-1 DO50J=1,NBU
     *S
C THE ADMITTANCE IS DIVIDES INTO REAL AND IMAGINARY PARTS.
      DO50I=1,NBUS G(I,J)=REAL(YBUS(I,J)) 50 B(I,J)=AIMAG(YBUS(I,J))
C VOLTAGE IS DIVIDED INTO REAL AND IMAGINARY PARTS.
      DO52I=1,NBUS ERAI(I)=REAL(VOLT(I)) J=NBUS+I 52 ERAI(J)=AIMAG(VOLT(
     *I)) N=2*NBUS NEQFJ=N-2
C REAL AND IMAGINARY VALUES OF CURRENT (CALCULATED) IS STORED.
      CUR(N)=0 CUR(NBUS)=0 67 DO54I=1,NBU SUM=CMPLX(0.,0.) DO55J=1,NBUS
      K=NBUS+J YA=CMPLX(ERAI(J),ERAI(K)) YB=CMPLX(G(I,J),B(I,J)) 55 SUM=
     *SUM+(YA*YB) CUR(I)=REAL(SUM) 54 CUR(I+NBUS)=AIMAG(SUM) IF(NFPCR.NE
     *.1)GOTO80 WRITE(LWE,53)(ERAI(I),I=1,NEQFJ) 53 FORMAT(2X,'ERAI',10F
     *10.4)
80    IF(NFPVO.NE.1)GOTO209 WRITE(LWE,302)(CUR(I),I=1,NEQFJ) 302 FORMAT(
     *2X,'CUR',5F12.6/(5X,5F12.6))
209   DO56L=1,NBU NA=NBUS+L IA=NA-1 RSUM=0. QSUM=0. DO57M=1,NBUS MA=M+NB
     *US REPRT=(ERAI(M)*G(L,M))-(ERAI(MA)*B(L,M)) RIPRT=(ERAI(MA)*G(L,M)
     *)+(ERAI(M)*B(L,M)) RSUM=RSUM+(ERAI(L)*REPRT)+(ERAI(NA)*RIPRT) QSUM
     *=QSUM+(ERAI(NA)*REPRT)-(ERAI(L)*RIPRT) 57 CONTINUE IF(IDENB(L).NE.
     *5)GOTO58 GIVEN(L)=PLD(L)-RSUM GIVEN(IA)=QLD(L)-QSUM GIVEA(L)=GIVEN
     *(L) GIVEA(IA)=GIVEN(IA) GOTO56 58 IF(ITERAT.EQ.1)ABSOL(L)=CABS(VOL
     *T(L)) GIVEN(L)=PLD(L)-RSUM GIVEA(L)=GIVEN(L) GIVEN(IA)=-((ERAI(L))
     ***2)-((ERAI(NA))**2)+(ABSOL(L))**2 GIVEA(IA)=GIVEN(IA) 56 CONTINUE
      IF(NFPGI.NE.1)GOTO210 WRITE(LWE,75)(GIVEA(I),I=1,NEQFJ) 75 FORMAT(
     *2X,'GIVEA',8F12.6/(7X,8F12.6))
C CALCULATION OF JACOBIAN.
  210 CONTINUE DO5500I=1,NBU K=I+NBUS L=K-1 DO5100J=1,NBU JPN=J+NBU JMTR
     *X(I,J)=ERAI(I)*G(I,J)+ERAI(K)*B(I,J) JMTRX(I,JPN)=ERAI(K)*G(I,J)-E
     *RAI(I)*B(I,J) 5100 CONTINUE JMTRX(I,I)=JMTRX(I,I)+CUR(I) JMTRX(I,L
     *)=JMTRX(I,L)+CUR(K) IF(IDENB(I).EQ.4)GOTO5300 DO5200J=1,NBU JPN=J+
     *NBU JMTRX(L,J)=ERAI(K)*G(I,J)-ERAI(I)*B(I,J) JMTRX(L,JPN)=-ERAI(K)
     **B(I,J)-ERAI(I)*G(I,J) 5200 CONTINUE JMTRX(L,I)=JMTRX(L,I)-CUR(K)
      JMTRX(L,L)=JMTRX(L,L)+CUR(I) GOTO5500 5300 CONTINUE DO5400J=1,NBU
      JPN=J+NBU JMTRX(L,J)=0 JMTRX(L,JPN)=0 5400 CONTINUE JMTRX(L,I)=2.*
     *ERAI(I) JMTRX(L,L)=2.*ERAI(K) 5500 CONTINUE IF(NFPAJ.NE.1)GOTO205
      WRITE(LWE,72)((JMTRX(I,J),J=1,NEQFJ),I=1,NEQFJ) 72 FORMAT(2X,'AJAC
```

```
      *',8F12.6/4(7X,8F12.6/))
                                                                              0
  205  ERS=1.E-14 CALLLNEQNS(JMTRX,40,40,NEQFJ,GIVEN,IR,IC,IER) WRITE(LWEO
      *,320)IER 320 FORMAT(2X,'IER',I15)
       DO66I=1,NBU J=I+NBUS J1=J-1 ERAI(J)=GIVEN(J1)+ERAI(J) 66 ERAI(I)=EO
      *RAI(I)+GIVEN(I) K=NEQFJ ITERAT=ITERAT+1 IF(ITERAT.GT.40)GOTO500 BLO
      *ARGE=ALARGE(K,GIVEA) WRITE(LWE,310)BLARGE 310 FORMAT(2X,'BLARGE',EO
      *20.8)
                                                                              0
       IF(BLARGE.GT..000100)GOTO67 351 DO68I=1,NBUS NA=I+NBUS VOLT(I)=CMPO
      *LX(ERAI(I),ERAI(NA)) 68 CONTINUE DO84I=1,NBUS IF(I.EQ.NBUS)GOTO85 O
       GOTO84 85 YA=VOLT(NBUS) VOLT(NBUS)=VOLT(LOVE) VOLT(LOVE)=YA          O
   84 CONTINUE LA=LOVE LB=NBUS DO89I=1,NBUS IF(I.NE.LA)GOTO89 DO86J=1,NBO
      *US SUM=YBUS(I,J) YBUS(I,J)=YBUS(LB,J) YBUS(LB,J)=SUM 86 CONTINUE   O
   89  CONTINUE DO90I=1,NBUS IF(I.NE.LA)GOTO90 DO87J=1,NBUS SUM=YBUS(J,I)O
       YBUS(J,I)=YBUS(J,LB) YBUS(J,LB)=SUM 87 CONTINUE 90 CONTINUE WRITE(O
      *LWE,761) 761 FORMAT(1H1,27X,'LINE FLOWS')
                                                                              0
       WRITE(LWE,762) 762 FORMAT(2X,'FROM BUS',2X,'TO BUS',4X,'REAL POWERO
      *',2X,'REACTIVEPOWER')
                                                                              0
       CLOSS=0. DO501I=1,NBUS SUM=CMPLX(0.,0.) DO502J=1,NLIN IF(IBFR(J).NO
      *E.I.AND.IBTO(J).NE.I)GOTO502 IF(IBFR(J).EQ.I)GOTO503 IA=IBTO(J) IBO
      *=IBFR(J) GOTO504 503 IA=IBFR(J) IB=IBTO(J) 504 YA=CONJG(VOLT(IA)-VO
      *OLT(IB)) IF(IDENA(J).EQ.1.AND.IA.EQ.IBFR(J))YA=CONJG((VOLT(IA)/STAO
      *RT(J))-VOLT(IB)) IF(IBTO(J).EQ.IA.AND.IDENA(J).EQ.1)YA=CONJG((VOLTO
      *(IA)*START(J))-VOLT(IB)) ZA=VOLT(IA) YB=CONJG(BIMP(J)) YG=(ZA*YA*YO
      *B) YAB1=CONJG(ZA) YAB2=CONJG(BUS(J)) YB=ZA*YAB1*YAB2 YG=YG+YB RLP=O
      *REAL(YG) REALP=AIMAG(YG) WRITE(LWE,763)(ALPA(JA,IA),JA=1,2),(ALPA(O
      *J1,IB),J1=1,2),RLP,REALP 763 FORMAT(2X,2A4,1X,2A4,1X,F10.6,2X,F10.O
      *6)
                                                                              0
  502 CONTINUE YA=VOLT(I) DO506J=1,NBUS YB=VOLT(J) 506 SUM=SUM+(YB*YBUS(O
      *I,J)) YG=CONJG(SUM) ZA=YA*YG RLP=REAL(ZA) PLD(I)=RLP CLOSS=CLOSS+RO
      *LP QLD(I)=AIMAG(ZA) 501 CONTINUE WRITE(LWE,764) 764 FORMAT(1H1,2X,O
      *'BUS',6X,'VOLT MAG',2X,'ANGLE DEG',2X,'REAL POWER',2X,'REACTIVE POO
      *WER')
                                                                              0
       DO765I=1,NBUS RLP=PLD(I) REALP=QLD(I) VAB=CABS(VOLT(I)) RA=REAL(VOO
      *LT(I)) RB=AIMAG(VOLT(I)) VAN=ATAN(RB/RA) PI=3.1416 VAN=180.*(VAN/PO
      *I) WRITE(LWE,766)(ALPA(J,I),J=1,2),VAB,VAN,RLP,REALP 766 FORMAT(2XO
      *,2A4,1X,F7.5,2X8F9.4,2X,F10.6,2X,F10.6)
                                                                              0
  765  CONTINUE ITERAT=ITERAT-1 WRITE(LWE,511)ITERAT,CLOSS 511 FORMAT(1H1O
      *,2X,'NUMBER OF ITERATIONS',I3,'LINE LOSS',F10.6)
                                                                              0
  500 STOP END FUNCTIONALARGE(K,GIVEA) DIMENSIONGIVEA(40) N=1 ALARGE=ABSO

      *(GIVEA(1)) DO1M=2,K SA=ABS(GIVEA(M)) IF(ALARGE.GT.SA)GOTO1 ALARGE=O
      *SA 1 CONTINUE RETURN END
                                                                              0
$ENTRY
```

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EBUG***** UNIVERSITY OF WATERLOO *****14.31.47****FRIDAY    **** 3 DEC 71****
 $JOB   WATFIV  T0628KES.BHAT    P=29
C      ****************************************************************
C       LOAD FLOW PROGRAM - YBUS - GAUSS-SEIDEL
C      ****************************************************************
      CHARACTER*8NAM(60) COMPLEXSI,LOSSN INTEGER*2IGEN(60),NUM(60),NZR(6
     *0,15) COMPLEXA(60),E(60),EC(60),S(60),SGEN(60),CP(60),YBUS(60,15) 0
      COMPLEXSUM,SUM1,Y,Z,YP,TR1,HALF,STOT COMPLEXCONJG,CMPLX REALQMIN(6
     *0),QMAX(60),AMG(60),AMAG(60) PD=45./ATAN(1.) HALF=0.5 ITAPE1=4 REW0
     *INDITAPE1
C      TITLE CARD IS READ.                                              0
      READ603,(NAM(I),I=1,10) PRINT1112,(NAM(I),I=1,10) READ34,ACCN,ERR0
     *R,ISL,IALT PRINT888,ISL PRINT33,ERROR PRINT39,ACCN IP=1 JP=1 KL=0 0
      READ9,N,M,IPP
C      IPP IS GREATER THAN 0 IF TOTAL LINE CHARGING CAPACITANCE IS GIVEN 0
C      KL = NUMBER OF GENERATORS EXCLUDING THE SLACK BUS
C      M = NUMBER OF LINES                                              0
      STOT=(0.0,0.0)                                                    0
C      READ THE NAMES FOR THE BUSES                                     0
      READ603,(NAM(I),I=1,N) PRINT609                                   0
C      INITIALISE THE YBUS MATRIX                                       0
      DO126I=1,N IGEN(I)=0 DO126J=1,15 126 YBUS(I,J)=(0.0,0.0) DO10I=1,N0
      READ604,K,SUM1,YP,SUM,Y,L S(K)=SUM1 E(K)=SUM YBUS(K,1)=YBUS(K,1)+Y0
     *P PRINT605,NAM(K),S(K),YP,E(K),Y
C      -P,-Q FOR LOAD AND P,Q FOR GENERATION                            0
      S(K)=CONJG(S(K)+Y) SGEN(K)=Y EC(K)=CONJG(E(K)) AMG(K)=CABS(E(K)) S0
     *TOT=STOT+S(K)
C      L IS GREATER THAN  SIX IF IT IS A GENERATOR BUS AND NEXT CARD HAS 0
C      THE MAGNITUDE, QMIN AND QMAX.                                    0
      IF(L.LE.6)GOTO10 KL=KL+1 IGEN(K)=1 READ930,AMAG(K),QMIN(K),QMAX(K)0
   10 CONTINUE DO91I=1,N NUM(I)=1 91 NZR(I,1)=I
C      READ THE LINE DATA                                               0
      PRINT610 DO99K=1,M READ2,I,J,Z,YP,TRATIO                          0
C      CALCULATE THE LINE ADMITTANCE                                    0
      Y=1./Z IF(IPP.GT.0)YP=YP*HALF                                     0
C    . DATA FOR PARALLEL LINES SHOULD APPEAR TOGETHER                   0
C*********************************************************************** 0
C
      IF((I.EQ.IP).AND.(J.EQ.JP))GOTO6111 IJ=NUM(I)+1 NUM(I)=IJ NZR(I,IJ0
     *)=J IJ1=NUM(J)+1 NUM(J)=IJ1 NZR(J,IJ1)=I IP=I JP=J 6111 CONTINUE I0
     *F(TRATIO.GT.0)GOTO5111 PRINT2,I,J,Z,YP Z=YP GOTO5112
 5111 CONTINUE PRINT2,I,J,Y,YP,TRATIO Y=1./TRATIO*Y SUM=Y*(1.-TRATIO) Z=0
     *YP-SUM YP=YP+SUM/TRATIO 5112 CONTINUE YBUS(I,1)=YBUS(I,1)+Y+YP YBU0
     *S(J,1)=YBUS(J,1)+Y+Z YBUS(I,IJ)=YBUS(I,IJ)-Y YBUS(J,IJ1)=YBUS(J,IJ0
     *1)-Y WRITE(ITAPE1,2)I,J,Y,YP,Z 99 CONTINUE STOT=
     *STOT-S(ISL) SI=STOT K2=0 PRINT3,K2 PRINT30,-SI 100 K2=K2+1 IPRNT=00
      SUM1=(0.,0.) DO12J=1,N SUM=(0.,0.) N1=NUM(J) DO11LI=2,N1 L=NZR(J,L0
     *I) 11 SUM=SUM+YBUS(J,LI)*E(L) CP(J)=SUM SUM=SUM+YBUS(J,1)*E(J) SUM0
     *=SUM*EC(J) SUM1=SUM1+SUM A(J)=SUM 12 CONTINUE IF(ISL.GT.0)S(ISL)=A0
     *(ISL) LOSSN=SUM1 IF(KL.LE.0)GOTO129 DO164J=1,N IF(IGEN(J).LE.0)GOT0
     *0164 SUM=A(J) SI=SI-S(J) AM=-AIMAG(SUM) IF(AM.GE.QMIN(J))GOTO593 A0
     *M=QMIN(J) PRINT595,J GOTO594 593 IF(AM.LE.QMAX(J))GOTO594 AM=QMAX(0
     *J) PRINT596,J 594 S(J)=CMPLX(REAL(S(J)),-AM) SI=SI+S(J) 164 CONTIN0
     *UE 129 CONTINUE
      S(ISL) = -SI + SUM1                                               0
      DO 425 I = 1,N                                                    0
      IF(K2 .LE. 1 .AND. I .EQ. ISL ) GO TO 425                         0
                                    IF(IALT.GT.0)GOTO422 IF(I.EQ.ISL)GOTO420
     *S 422 CONTINUE SUM=(0.,0.) A(I)=(S(I)/EC(I)-CP(I))/YBUS(I,1)      0
  423 CONTINUE SUM1=A(I)-E(I) SUM=SUM1*ACCN IF(IGEN(I).LE.0)GOTO42 SUM1=0
     *E(I)+SUM SUM=SUM1/CABS(SUM1)*AMAG(I)-E(I) 42 E(I)=E(I)+SUM EC(I)=C0
     *ONJG(E(I)) N1=NUM(I) DO424J1=2,N1 J=NZR(I,J1) IF(J.GT.I)CP(J)=CP(J0
```

```
     *)+YBUS(I,J1)*SUM 424 CONTINUE IF(ABS(CABS(A(I))-AMG(I)).GT.ERRO
     *PRNT=1 AMG(I)=CABS(E(I)) 425 CONTINUE
      PRINT 711,(E(I),I=1,N)
  711 FORMAT(6(2F8.5,2X))

                                       IF(IPRNT.GT.0)GOTO100 PRINT30
     *,K2 PRINT526,LOSSN REWINDITAPE1 PRINT607 D0333I=1,N AN=ATAN(AIMAG(
     *E(I)/REAL(E(I))))*PD SUM=CONJG(S(I)) PRINT606,NAM(I),E(I),AMG(I),A
     *N,SUM 333 CONTINUE PRINT600 PRINT601 D0504I=1,N D0503J=1,M READ(
     *APE1,2)I1,J1,Y,YP,Z IF(I1.NE.I)GOTO501 SUM=CONJG((E(I1)-E(J1))*Y)
      SUM=E(I1)*(SUM+CONJG(E(I1)*YP)) PRINT602,NAM(I1),NAM(J1),SUM GOTO50
     *03 501 IF(J1.NE.I)GOTO503 SUM=CONJG((E(J1)-E(I1))*Y) SUM=E(J1)*(SU
     *M+CONJG(E(J1)*Z)) PRINT602,NAM(J1),NAM(I1),SUM 503 CONTINUE
  504 REWINDITAPE1 PRINT608 2 FORMAT(2I5,6F10.6)
    3 FORMAT(1H0,'  ITERATION =',I3,/)
    9 FORMAT(5I5)
   30 FORMAT(' CONJG(SLACK BUS POWER - NETWORK LOSS) = ',2F10.5)
   33 FORMAT('    *** ACCURACY OF VOLTAGE MAGNITUDE',F8.5)
   34 FORMAT(2F10.5,12I5/12I5)
   39 FORMAT(10X,'********ACCELERATION FACTOR=',F5.2)
  526 FORMAT(' CONJG(TRANSMISSION NETWORK LOSS) =',2F10.5)
  595 FORMAT(' GENERATOR BUS',I4,' IS CONVERTED TO LOAD BUS WITH QMIN.')
  596 FORMAT(' GENERATOR BUS',I4,' IS CONVERTED TO LOAD BUS WITH QMAX.')
  600 FORMAT(1H1,//,10X,'*****LINE FLOWS*****',//)
  601 FORMAT('  FROM BUS  TO BUS',6X,'REAL POWER  REACTIVE POWER'/)
  602 FORMAT(2X,A8,1X,A8,2F15.6)
  603 FORMAT(10A8)
  604 FORMAT(I5,4F10.4,2F5.3,2F10.4,I5)
  605 FORMAT(1X,A8,4F10.4,2F5.2,2F10.4)
  606 FORMAT(1X,A8,6F10.5)
  607 FORMAT(1H1,1X,'BUS NAME',19X,'BUS VOLTAGE',14X,'BUS POWER'/14X,'RE
     *AL  IMAGINARY  MAGNITUDE    ANGLE',5X,'REAL   REACTIVE'/)
  608 FORMAT(1H1)
  609 FORMAT(///'          BUS DATA'/)
  610 FORMAT(///'          LINE DATA'/)
  888 FORMAT(/10X,' SLACK BUS              ',I0I3/)
  930 FORMAT(3F10.5)
 1112 FORMAT(1H1,20X,10A8//)
      STOP END
$ENTRY
```

```
***DEBUG***** UNIVERSITY OF WATERLOO *****12.53.56****FRIDAY   **** 3 DE
      $JOB     WATFIV   G0000UON,KP=29             PAUL KISH  ELEC. ENG.
      C*****************************************************************
      C      TRANSIENT STABILITY PROGRAM
      C*****************************************************************
             COMPLEX YBUS(40,40),YBL(40,40),REACT(12),V(25)
             COMPLEX IMP,YP,Y,SP,VV(12),YD(50)
             COMPLEX CMPLX,CONJG
             REAL*8 PMI(12),A(12,12),THETA(12,12),MCONST(12)
             REAL*8 D(25),W(25),DY(25)
             REAL*4 P(25),Q(25),AD(12),PP(25),QQ(25)
             REAL*4 CABS
             INTEGER CARD(20),G(25),REF(50)
             COMMON YBUS,YBL,YD,A,THETA,PMI,MCONST,NN
             EXTERNAL VECTOR
           2 FORMAT(20A4)
           3 FORMAT('0',20A4)
           4 FORMAT(10X,I2,1X,I2,1X,I2)
           6 FORMAT(I2,3X,4F10.5,I2,3X,2F10.5)
          10 FORMAT(I2,3X,3F10.5)
          11 FORMAT(I2,8X,2(I2,3X),4F10.5)
          20 FORMAT(4(I2,3X),F10.5)
          80 FORMAT(3F10.5)
         301 FORMAT('0','INITIAL LOAD FLOW',/' ','BUS NUMBER',12X,
            1'BUS VOLTAGE',5X,'PL',8X,'QL',8X,'PG',8X,'QG')
         302 FORMAT(' ',5X,I2,6X,6F10.5)
         303 FORMAT('0','INPUT MACHINE DATA',/' ','BUS NUMBER',3X,
            1'INERTIA CONST.',3X,'D.A. TRANS. REACT.')
         304 FORMAT(' ',5X,I2,8X,F10.5,5X,2F10.5)
         305 FORMAT('0','INPUT LINE DATA',/' ','LINE NO.',3X,'FROM BUS',
            13X,'TO BUS',10X,'SERIES IMP.',9X,'CHARGING ADMIT.')
         306 FORMAT(' ',4X,I2,9X,I2,7X,I2,5X,4F10.5)
         311 FORMAT('0',' YBUS - N+M TERMINAL REPRESENTATION')
         312 FORMAT(' ',I2,5(8(2F8.2),/' '))
         307 FORMAT('0',' YBLF - M TERMINAL REPRESENTATION')
         308 FORMAT(' ',I2,2(8(2F8.2),/' '))
         315 FORMAT('0','FAULT ON LINE NO.',1X,I2,2X,'AT A DIST. OF',1X,F5.
            12X,'FROM BUS',1X,I2,2X,'***FAULT CLEARED AT TC=',1X,F6.3,1X,'SE
            1)
         309 FORMAT('0',' YBLPF - M TERMINAL REPRESENTATION')
         608 FORMAT('0',' YBL - M TERMINAL REPRESENTATION')
         638 FORMAT('0',' LOAD DROPPED OFF BUS NO.'1X,I2)
          92 FORMAT('0',' SWING CURVES FOR T LT TC',//' ',
            1'TIME',20X,'ANGLE',/)
          93 FORMAT('0','SWING CURVES FOR T GT TC',/)
          91 FORMAT(' ',F6.4,4X,12F9.3)
      C*****************************************************************
      C      HEADING
      C*****************************************************************
             READ(5,2)CARD
             WRITE(6,3)CARD
      C*****************************************************************
      C      SYSTEM DIMENSIONS
      C*****************************************************************
             READ(5,4)N,NN,L
             NT=N+NN
      C*****************************************************************
      C      INITIALIZE YBUS
      C*****************************************************************
             DO 9 I=1,NT
             DO 9 J=1,NT
           9 YBUS(I,J)=(0.00,0.00)
```

```
      YP=(0.00,0.00)
C*************************************************************************
C     INPUT LOAD FLOW
C*************************************************************************
      WRITE(6,301)
      DO 5  I=1,N
      READ(5,6)J,P(J),Q(J),V(J),G(J),PP(J),QQ(J)
      WRITE(6,302)J,V(J),P(J),Q(J),PP(J),QQ(J)
      IF(I.GT.NN) GO TO 5
      PMI(I)=PP(I)
    5 CONTINUE
C*************************************************************************
C     INPUT MACHINE DATA
C*************************************************************************
      WRITE(6,303)
      DO 16  I=1,NN
      READ(5,10)J,MCONST(J),REACT(J)
      WRITE(6,304)J,MCONST(J),REACT(J)
      Y=(1.00,0.00)/REACT(J)
      NBUSI=J
      NBUSJ=NBUSI+NN
   16 CALL YBUSAL(NBUSI,NBUSJ,Y,YP)
C*************************************************************************
C     INPUT LINE DATA
C*************************************************************************
      WRITE(6,305)
      DO 17  I=1,L
      READ(5,11)LL,II,JJ,IMP,YP
      WRITE(6,306)LL,II,JJ,IMP,YP
      Y=(1.00,0.00)/IMP
      NBUSI=II+NN
      NBUSJ=JJ+NN
      YD(LL)=Y
   17 CALL YBUSAL(NBUSI,NBUSJ,Y,YP)
C*************************************************************************
C     CONVERT LOADS TO EQUIV. ADMITTANCES
C*************************************************************************
      DU 14  I=1,N
      IF(G(I).EQ.0) GO TO 14
      SP=CMPLX(P(I),-Q(I))
      J=I+NN
      YBUS(J,J)=YBUS(J,J)+SP/(V(I)*CONJG(V(I)))
   14 CUNTINUE
C*************************************************************************
C     COMPUTE GENERATOR INTERNAL VOLTAGES
C*************************************************************************
      DU 15  I=1,NN
      SP=CMPLX(PP(I),-QQ(I))
   15 VV(I)=V(I)+REACT(I)*SP/CONJG(V(I))
C*************************************************************************
C     FAULT SPECIFICATION
C*************************************************************************
      READ(5,20)KEY,LL,JK,JKK,F
      IF(KEY.EQ.3) WRITE(6,638)JK
C     *******************************************************************
C     N+M TERMINAL REPRESENTATION
C     *******************************************************************
      WRITE(6,311)
      DO 157  I=1,NT
  157 WRITE(6,312)I,(YBUS(I,J),J=1,NT)
C*************************************************************************
C     M TERMINAL REPRESENTATION
C*************************************************************************
```

```
      CALL YBLF(KEY,LL,JK,JKK,F,NT,MO)
      CALL SUPRES(N,MO)
C***********************************************************************
C     SPEC. TIME INCREMENT,CLEARING TIME,RUNNING TIME
C     NB CLEARING TIME SOME MULTIPLE OF TIME INCREMENT
C     NB CLEARING TIME NOT EQUAL TO RUNNING TIME FOR FAULTS
C     NB CLEARING TIME EQUAL TO RUNNING TIME FOR LOAD SHEDDING
C***********************************************************************
      READ(5,80)DT,TC,TCC
      TC=TC-0.00001
      IF(KEY.EQ.3) WRITE(6,608)
      IF(KEY.EQ.3) GO TO 609
  401 WRITE(6,315)LL,F,JK,TC
      WRITE(6,307)
  609 DO 407  I=1,NN
  407 WRITE(6,308)I,(YBL(I,J),J=1,NN)
C***********************************************************************
C     PREPARE PARAMETERS USED IN D.E.'S
C***********************************************************************
      T=0.00
      NN2=2*NN
   83 DO 71  I=1,NN
      DO 70  J=1,NN
      IF(I.EQ.J)A(I,J)=CABS(VV(I))*CABS(VV(I))*REAL(YBL(I,I))
      IF(I.EQ.J) GO TO 70
      A(I,J)=CABS(VV(I))*CABS(VV(J))*CABS(YBL(I,J))
      IF(REAL(YBL(I,J)).EQ.0.0.AND.AIMAG(YBL(I,J)).EQ.0.0) THETA(I,J
     100
      IF(REAL(YBL(I,J)).EQ.0.0.AND.AIMAG(YBL(I,J)).EQ.0.0)  GO TO 70
      THETA(I,J)=ATAN2(AIMAG(YBL(I,J)),REAL(YBL(I,J)))
   70 CONTINUE
      IF(TC.EQ.TCC) GO TO 71
      D(I)=ATAN2(AIMAG(VV(I)),REAL(VV(I)))
      JJ=I+NN
      D(JJ)=0.00
   71 CONTINUE
C***********************************************************************
C     COMPUTE SWING CURVES
C***********************************************************************
      IF(T.EQ.0.0) WRITE(6,92)
      IF(TC.EQ.TCC) WRITE(6,93)
      DO 81  I=1,200
   76 DO 75  J=1,NN
   75 AD(J)=D(J)*180.0/3.14159
      WRITE(6,91)T,(AD(J),J=1,NN)
      IF(T.GE.TC.AND.TC.NE.TCC) GO TO 82
      CALL RK5ES(T,D,DY,DT,NN2,VECTOR)
      IF(T.GE.TC.AND.TC.NE.TCC) GO TO 76
      IF(T.GE.TCC.OR.I.EQ.200) STOP
   81 CONTINUE
   82 TC=TCC
      IF(KEY.EQ.3) STOP
C***********************************************************************
C     COMPUTE YBLPF
C***********************************************************************
      DO 85  I=1,NT
      DO 85  J=1,NT
   85 YBL(I,J)=YBUS(I,J)
      IF(KEY.EQ.2) GO TO 89
      CALL YBLPF(JK,JKK,LL)
   89 MO=NT
      CALL SUPRES(N,MO)
      WRITE(6,309)
```

```
      DO 86  I=1,NN
   86 WRITE(6,308)I,(YBL(I,J),J=1,NN)
      GO TO 83
      END
C***********************************************************************
      SUBROUTINE SUPRES(N,MO)
C***********************************************************************
      COMPLEX YBL(40,40),YBUS(40,40),YD(50)
      COMPLEX B
      REAL*8 A(12,12),THETA(12,12),PMI(12),MCONST(12)
      COMMON YBUS,YBL,YD,A,THETA,PMI,MCONST,NN
      M=MO-NN
      DO 30  I=1,M
      II=MO-I
      JJ=MO-I+1
      DO 30  J=1,II
      DO 30  JS=1,II
      B=YBL(J,JJ)*YBL(JJ,JS)/YBL(JJ,JJ)
   30 YBL(J,JS)=YBL(J,JS)-B
      RETURN
      END
C***********************************************************************
      SUBROUTINE YBUSAL(NBUSI,NBUSJ,Y,YP)
C***********************************************************************
      COMPLEX YBL(40,40),YBUS(40,40),YD(50)
      REAL*8 A(12,12),THETA(12,12),PMI(12),MCONST(12)
      COMPLEX Y,YP
      COMMON YBUS,YBL,YD,A,THETA,PMI,MCONST,NN
      YBUS(NBUSI,NBUSI)=Y+YBUS(NBUSI,NBUSI)+YP
      YBUS(NBUSJ,NBUSJ)=Y+YBUS(NBUSJ,NBUSJ)+YP
      YBUS(NBUSI,NBUSJ)=YBUS(NBUSI,NBUSJ)-Y
      YBUS(NBUSJ,NBUSI)=YBUS(NBUSI,NBUSJ)
      RETURN
      END
C***********************************************************************
      SUBROUTINE YBLPF(JK,JKK,LL)
C***********************************************************************
      COMPLEX YBL(40,40),YBUS(40,40),YD(50)
      COMPLEX Y
      REAL*8 A(12,12),THETA(12,12),PMI(12),MCONST(12)
      COMMON YBUS,YBL,YD,A,THETA,PMI,MCONST,NN
      Y=YD(LL)
      II=JK+NN
      JJ=JKK+NN
      YBL(II,II)=YBL(II,II)-Y
      YBL(JJ,JJ)=YBL(JJ,JJ)-Y
      YBL(II,JJ)=YBL(II,JJ)+Y
      YBL(JJ,II)=YBL(II,JJ)
      RETURN
      END
C***********************************************************************
      SUBROUTINE YBLF(KEY,LL,JK,JKK,F,NT,MO)
C***********************************************************************
      COMPLEX YBL(40,40),YBUS(40,40),YD(50)
      COMPLEX Y,CMPLX
      REAL*8 A(12,12),THETA(12,12),PMI(12),MCONST(12)
      COMMON YBUS,YBL,YD,A,THETA,PMI,MCONST,NN
      IF(KEY.EQ.1)GO TO 200
      IF(KEY.EQ.3) GO TO 203
```

```
      DO 25  I=1,NT
      DO 25  J=1,NT
      II=I
      JJ=J
      IF(J.GE.(JK+NN))JJ=J+1
      IF(I.GE.(JK+NN))II=I+1
      IF(JJ.GT.NT.OR.II.GT.NT) GO TO 25
      YBL(I,J)=YBUS(II,JJ)
   25 CONTINUE
      MO=NT-1
      RETURN
  200 Y=YD(LL)
  203 CONTINUE
      DO 201  I=1,NT
      DO 201  J=1,NT
  201 YBL(I,J)=YBUS(I,J)
      IF(KEY.EQ.3) GO TO 202
      II=JK+NN
      JJ=JKK+NN
      T=1.0/F
      TT=1.0/(1.0-F)
      YBL(II,JJ)=YBL(II,JJ)+Y
      YBL(JJ,II)=YBL(II,JJ)
      YBL(II,II)=YBL(II,II)-Y+CMPLX(T,0.0)*Y
      YBL(JJ,JJ)=YBL(JJ,JJ)-Y+CMPLX(TT,0.0)*Y
  202 MO=NT
      RETURN
      END
C******************************************************************
      SUBROUTINE VECTOR(X,Y,W,NN2)
C******************************************************************
      COMPLEX YBL(40,40),YBUS(40,40),YD(50)
      REAL*8 A(12,12),THETA(12,12),PMI(12),MCONST(12)
      REAL*8 X,Y(25),W(25)
      REAL*8  DCOS
      COMMON YBUS,YBL,YD,A,THETA,PMI,MCONST,NN
      DO 72  I=1,NN
      L=I+NN
      W(L)=PMI(I)-A(I,I)
      DO73  J=1,NN
      IF(J.EQ.I) GO TO 73
      W(L)=W(L)-A(I,J)*DCOS(THETA(I,J)-(Y(I)-Y(J)))
   73 CONTINUE
      W(L)=W(L)/MCONST(I)
      W(I)=Y(L)
   72 CONTINUE
      RETURN
      END
$ENTRY
```